Contravariant power constructions

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Power constructions are widely considered in the literature for many categories. In the category of sets, the powerset construction gives rise to two functors: a covariant one sending a function $f : S \to T$ to the direct image function $f_* : \mathcal{P}(S) \to \mathcal{P}(T)$, and a contravariant one which sends a function $f : S \to T$ to the preimage function $f^* : \mathcal{P}(T) \to \mathcal{P}(S)$. While the former is known to lift to several categories, e.g. for compact Hausdorff topological spaces in the form of Vietoris construction, and for universal algebras with the power algebra equipped with pointwise operations, the same is not true for the latter. For instance, in the category of compact Hausdorff topological spaces, for f^* to be continuous, it is not sufficient to require that f is continuous, and a similar phenomenon holds for algebras.

In this talk I will show that, under some conditions, contravariant power constructions are well-behaved for compact Hausdorff topological spaces and for free semigroups. This provides a very simple type theoretic analysis of (existential) quantifiers as spans in various settings including logic on words.

This is based on joint work with Mai Gehrke.