Lax comma 2-categories

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The categorical Galois theory, originally developed by Janelidze [9, 1, 10], gives a unifying setting for most of the formerly introduced Galois type theorems [1], even generalizing most of them. It neatly gives a common ground for Magid's Galois theory of commutative rings [14, 7, 2], Grothendieck's theory of étale covering of schemes, and central extension of groups (see Chapter 5 of [1]). Furthermore, Janelidze's Galois theory has found several developments, applications and examples in new settings since its introduction [1, 8, 6, 4, 15, 11, 13].

The most elementary observation on factorization systems and Janelidze-Galois theory is that, in the suitable setting of finitely complete categories, the notion of absolute admissible Galois structure coincides with that of a semi-left exact reflective functor/adjunction [3, 1].

Our original aim was to get 2-dimensional analogues for the basic concepts (and results) of absolute Galois theory. As a guinding template, we used the fact mentioned above and the theory of simple 2-monads developed in [5]. Therefore our first step was to develop a suitable counterpart notion to that of semi-left exact reflective functor compatible with the notion of simple 2-adjunctions of [5].

At this point, the notion of *lax comma* 2*-categories* comes into play as a fundamental aspect of our work. Even being a recurrent notion in the literature, we couldn't find a systematic study covering the understanding we needed to suitably develop our theory.

Among the basic aspects on lax comma 2-categories, we have the following: they can be defined as *Gray*-limits, they have several change of base functors going on, and they are "simple" examples of 2-categories of lax algebras (and hence we could use as our basic tool the theorems of [12]).

The main aim of this talk is to establish some of these facts on lax comma 2-categories.

This is joint work with Maria Manuel Clementino.

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