The assembly as a free construction and an application to bitopological categories

Pointfree topology regards certain order-theoretical structures, called *frames*, as abstract topological spaces. Frames equipped with certain morphisms form a category called **Frm**. We may see these as abstract topological spaces in virtue of an important adjunction $\Omega : \mathbf{Frm}^{op} \leftrightarrows \mathbf{Top} : \mathsf{pt}$, in which the functor pt assigns to each frame something akin to a Zariski spectrum.

I will introduce the assembly of a frame L. This is an object $\mathsf{A}(L)$ of **Frm** which may be characterized in two ways.

1. In terms of a universal property: we have a map $\nabla : L \to \mathsf{A}(L)$ universal with the property that each $\nabla(x)$ is complemented.
2. As the collection of all frame congruences on L.

I will then show how one can see the assembly as a free frame.

I will then briefly introduce and motivate the notion of *bitopological space*: a set equipped with two topologies. Bitopological spaces with *bicontinuous maps* form a category called **BiTop**. There are two categories which serve as an abstraction of **Bitop**, and these are the categories **dFrm** of d-frames, and **BiFrm** of biframes. In both cases we have a suitable contravariant adjunction analogous to $\Omega \dashv \mathsf{pt}$.

I will explain how our description of the assembly as a free frame allows us to generalize the notion to both **dFrm** and **BiFrm**, so that we can obtain objects with the universal property (1) in both these categories. I will briefly mention why the characterization (2) is not suitable anymore in the bitopological setting.