

## Abstract:

*Actions and commutators via cross effects of functors: modules, crossed modules and a new notion of commutator*

Manfred Hartl

This is an extension of the talk of B. Loiseau, in the setting of now mostly semi-abelian categories  $\mathbb{C}$ : we first give several characterizations of Beck modules in  $\mathbb{C}$ , which allows to characterize extensions (= short exact sequences) in  $\mathbb{C}$  whose conjugation action on the kernel factors through a module structure over the cokernel. In particular, we explicitly construct a left adjoint of the embedding of the category of such extensions into the category of all extensions in  $\mathbb{C}$ . This also amounts to a simple construction of a left-adjoint of the embedding of the category of  $G$ -modules into the category of  $G$ -actions. We then give a characterization of crossed modules in  $\mathbb{C}$  in terms of cross-effects, and use it to establish some fundamental properties.

Cross-effects also allow to introduce a new commutator of a collection of  $n$  subobjects of a given object in  $\mathbb{C}$ . For  $n = 2$  it coincides with the normal closure of the Huq-commutator, but for  $n > 2$  it is substantially bigger than the latter. On the other hand, it has various good properties, among others a nice distributivity law with respect to joins. Moreover, there is a strong algebraic structure related to the lower central series defined by the new commutator: the collection of its successive quotients, viewed as a graded abelian object, is a graded algebra over an operad associated with  $\mathbb{C}$ . This is work in progress and will only be outlined in the end.