A Schreier-Mac Lane extension theorem in action accessible categories

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Joint work with Dominique Bourn

The aim of this talk is to give an intrinsic version of the Schreier-Mac Lane extension theorem, classically known for groups ([4] IV, Theorems 8.7 and 8.8). Any extension of groups:

$$0 \longrightarrow K \xrightarrow{k} X \xrightarrow{f} Y \longrightarrow 0$$

determines, via conjugation in the group X, a group homomorphism $\phi : Y \longrightarrow \frac{\text{Aut } K}{\text{Int } K}$, called the *abstract kernel* of the extension. On the set $\text{Ext}_\phi(Y, K)$ of isomorphism classes of extensions with abstract kernel ϕ there is a simply transitive action of the abelian group $\text{Ext}_{\overline{\phi}}(Y, ZK)$, where ZK denotes the center of K and $\overline{\phi}$ is given by restricting the automorphism $\phi(y)$ to ZK.

In [2], Bourn showed that this result fully holds in any semi-abelian category $\mathbb{C}$ with *split extension classifier* [1]. This setting, however, excludes many interesting algebraic structures, like the category of rings. In the present talk we show that the same Schreier-Mac Lane extension theorem holds in a much wider class of categories, called *action accessible categories* [3], provided they are exact. This family of categories includes, beyond groups and Lie algebras, the categories of rings, associative algebras, Poisson algebras, Leibniz algebras, associative dialgebras (in the sense of J.L. Loday) and any variety of groups.

References

[1] F. Borceux, D. Bourn, *Split extension classifier and centrality*, Contemporary Mathematics, vol. 431 (2007), 85-104.

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[3] D. Bourn, G. Janelidze, *Centralizers in action accessible categories*, Cah. Topol. Gom. Diffr. Catg. 50 (2009), no. 3, 211-232.

[4] S. Mac Lane, *Homology*, Springer-Verlag, 1963.