THE EPIMORPHISM GAME FOR COMPLETELY REGULAR FRAMES

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Epimorphisms in completely regular frames are deeply mysterious. Leaving aside unsatisfactory descriptions involving abstract nonsense, the Lindelöf case is the only situation in which we have some sort of elementwise characterization. Even then the proof involves an excursion through **W**, the category of archimedean lattice-ordered groups with weak order unit.

In this talk we exhibit a a simple two-person game played on a frame extension $B \leqslant A$ with the feature that the game has a winning strategy for the second player iff B is epically embedded in A. Play takes place on a *game tree*, a subset $\{a_t : t \in T\} \subseteq A$ indexed by a tree T having the attributes listed below. (For $t < \top$ in T, we denote by t' the unique member of T satisfying $t \leqslant t' \lhd \top$. Here $t' \lhd \top$ means that there is no node strictly between t' and $\top$.)

1. Each $a \in A$ is of the form a_t for some index t of depth 1 in T.
2. For each non-leaf index t of even depth in T, $a_t = \bigvee_{r \lhd t} a_r$.
3. For each index t of odd depth, $\{a_r : r \lhd t\} = \{b \wedge a_t : a_{t'} \leqslant b \in B\}$.

Play goes like this.

1. Player 1 begins by choosing an element $a_0 \in A$.
2. Player 2 responds by choosing an upper bound b_0 for a_0 in B.
3. Player 1 answers by choosing an element a_1 from a prescribed subset of A which joins to b_0.
4. Player 2 responds by choosing another upper bound b_1 for a_0 in B.
5. Player 1 responds by choosing an element a_2 from a prescribed subset of A which joins to $b_1 \wedge a_1$.
6. Player 1 loses if she chooses an element $a_n \leqslant a_0$, otherwise she wins.

Theorem 1. *A subframe $B \leqslant A$ is epically embedded iff player 2 has a winning strategy in the epimorphism game played on some game tree for the extension.*

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