Normal and perfect locales

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In this talk we will look at the counterpart of the classical notion of perfectness in the category of locales. In notable contrast with the spatial case, this is not a self-dual concept. We give a notion of perfectness w.r.t. a fixed class of complemented sublocales that combined with the corresponding condition of normality allows to unify various types of perfect normality in frames.

The rôle of perfectness in the insertion problem on real functions is then described. Specifically, normality produces the standard (*weak*) *insertion* of a continuous real function h in between any upper semicontinuous f and any lower semicontinuous g satisfying $f \leq g$, while perfectness allows for the (*double*) *insertion* of an upper $\hat{f}$ and a lower $\hat{g}$ in between $f \leq g$ in such a way that $f < \hat{f} < \hat{g} < g$ whenever $f < g$. Combined, they yield (and, in fact, characterize) strict insertion of functions (providing, in particular, an extension of the well-known classical setting due to E. Michael):

perfectness + normality = double insertion + weak insertion = strict insertion

By dualizing these results, one gets a parallel picture for the case $f \geq g$:

co-perfectness + extremal disconnectedness = double insertion + weak insertion = strict insertion

(This is joint work in progress with J. Gutiérrez García and T. Kubiak.)