Descent and effective descent in the category of directed complete posets

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The category **Dcpo** of directed complete partial order sets plays an important role in domain theory and in theoretical computer science. This category is complete and cocomplete, but while limits can be easily described via the embedding of **Dcpo** in **Pos** (partial order sets and monotone maps), it is quite difficult to describe colimits. Fiech, in [1] characterizes and describes colimits in **Dcpo**, but his construction is rather complicated.

To give an easier characterization of colimits in **Dcpo** we use a result of Zhao Fan ([4]) and Keimel - Lawson ([3]). In these papers they proved that **Dcpo** is an epireflective subcategory of $\mathbf{Pos_T}$ (partial order sets endowed with the Scott topology and continuous maps). We characterize regular epimorphisms in $\mathbf{Pos_T}$ as the topological quotient maps, w.r.t. the Scott topology. As a consequence we find that regular epimophisms in **Dcpo** are topological quotient maps followed by a reflective embedding. Moreover helped by results of Janelidze and Sobral in [2] we characterize the descent and the effective descent morphisms in **Dcpo**.

References

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