Interior operators and topological connectedness

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The notion of interior operator introduced in an arbitrary category by Vorster ([4]) is used in the category of topological spaces. More precisely, notions of discrete and indiscrete objects with respect to an interior operator are introduced. These concepts can be used to construct two Galois connections between interior operators on the category **Top** and subclasses of topological spaces. These two Galois connections can be composed to create a third one that can be characterized via the notion of constant morphism. This gives rise to a commutative diagram of Galois connections. As a consequence, general notions of connectedness and disconnectedness with respect to an interior operator are introduced. The above mentioned commutative diagram of Galois connections is used to relate these new notions to Arhangelskii and Wiegandt's notions of connectedness and disconnectedness introduced in [1]. This shows that at least in topology, the notion of interior operator is just as good as the one of closure operator (cf. [2], [3]) to deal with appropriate general notions of connectedness and disconnectedness. Examples that illustrate the above theory are provided.

References

[1] A. V. Arhangel'skii and R. Wiegandt, *Connectedness and disconnectedness in topology*, Gen. Top. Appl. 5 (1975) 9–33.

[2] G. Castellini and D. Hajek, *Closure operators and connectedness*, Topology and its Appl. 5 (1994) 29–45.

[3] M. M. Clementino and W. Tholen, *Separation versus connectedness*, Topology and its Appl. 75 (1997) 143–181.

[4] S. J. R. Vorster, *Interior operators in general categories*, Quaestiones Mathematicae 23 (2000) 405–416.

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