

Separating families of locale maps and localic embeddings

Javier Gutiérrez García*

We exhibit a localic analogue of a family of maps that *separates points from closed sets*. In point-set topology, the latter is a crucial thing allowing the embedding of one space into another, especially into products. With our localic concept at hand, we are able to give in pointfree topology the localic version of the general topological embedding theorem (also called the *diagonal theorem*, cf. [1]).

Then a characterization of complete regularity of locales in terms of the localic unit interval is given, and the Johnstone-Tychonoff's embedding theorem for locales [2] is revisited by involving separating families.

Finally, assuming the axiom of choice, our general localic embedding theorem allows us to easily control the amount of factors of the target localic product which depends on the weight of the embeddable locale.

REFERENCES

- [1] R. Engelking, *General Topology*, Polish Sci. Publ., Warszawa, 1977.
- [2] P.T. Johnstone, *Stone Spaces*, Cambridge Univ. Press, Cambridge, 1982.

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