Linear Simulations?

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The concept of (bi)simulation plays an important role in computer science where it serves to compare labeled transition systems (LTSs). In order to "categorify" simulations, one first needs to formulate LTSs in categorical terms. Simulations should then act as morphisms between these.

LTSs can be viewed from two different angles:

- as faithful graph morphisms into some label-graph $\mathcal{X}$, we call these "processes";
- or as graph-morphisms from $\mathcal{X}$ into the bicategory ***rel*** of sets, relations and inclusions, we call these "systems".

The faithfulness of processes, respectively, the occurrence of ***rel*** as a base for systems correspond to the traditional computer-science requirement that parallel transitions should have different labels.

Simulations between systems can be viewed as lax transforms. These translate into certain spans of graph-morphimsm between processes (which motivated other attempts at categorification, via "open maps" or via coalgebra). When restricting to categories and functors, rather than graph morphisms, simulations between processes occupy a position between functors and profunctors over the label-category $\mathcal{X}$.

Dropping the restriction on parallel transitions results in a "resource-conscious" or "linear" notion of LTSs. The dichotomy between processes and systems carries over to this situation: ***rel*** as base is replaced by the bicategory ***spn*** of sets, spans and span-morphisms, while processes no longer need to be faithful. Depending on the choice of 1- and 2-cells, this essentially bijective correspondence extends to various biadjunctions we collectively refer to as "graph comprehension".

Transfering the ***rel***-based notion of simulation to the ***spn***-based context at the level of systems and at the level of processes somewhat surprisingly gives results that are not equivalent under graph-comprehension. In particular, lax transforms between ***spn***-based systems are insufficient to capture the result of the translation at the level of processes. One needs to employ certain *modules* for this purpose.

When restricting to (not necessarily faithful) functors into a label-category $\mathcal{X}$, we therefore find more candidates for the notion of "linear simulation" as a new kind of morphism over $\mathcal{X}$ that sits between functors and profunctors than in the ***rel***-based setting.