# Correspondences

Austin Melton

In 1982 H. Crapo introduced connections as generalizations of Galois connections. In 1990 H. Herrlich and M. Huŝek introduced Galois correspondences as concrete categorical generalizations of Galois connections. In this paper, we introduce correspondences as generalizations of both connections and Galois correspondences. We establish properties of correspondences, and we give an extended example. The example is a correspondence between the category of relations with order-continuous functions and the category of pretopological spaces with convergence-related functions.

In [1] Crapo defines a connection as follows. Let $(P, \leq)$ and $(Q, \leq)$ be partially ordered sets, and let $f : P \to Q$ and $g : Q \to P$ be order-preserving maps. $(f, P, Q, g)$ or simply $(f, g)$ is said to be a *connection* if $f$ and $g$ are quasi-inverses, i.e., if $fgf = f$ and $gfg = g$.

In [2] H. Herrlich and M. Huŝek generalize Galois connections to concrete categories and concrete functors; they call these generalizations Galois correspondences. Their defintion follows. Let $(\mathbf{A}, U)$ and $(\mathbf{B}, V)$ be concrete categories over $\mathbf{X}$, and let $G : A \to B$ and $F : B \to A$ be concrete functors over $\mathbf{X}$. $(G, (\mathbf{A}, U), (\mathbf{B}, V), F)$ or simply $(G, F)$ is said to be a *Galois correspondence* if $FG \leq 1_{\mathbf{A}}$ and if $1_{\mathbf{B}} \leq GF$.

Let $(\mathbf{A}, U)$ and $(\mathbf{B}, V)$ be concrete categories over $\mathbf{X}$, and let $G : A \to B$ and $F : B \to A$ be concrete functors over $\mathbf{X}$. $(G, (\mathbf{A}, U), (\mathbf{B}, V), F)$ or simply $(G, F)$ is said to be a *correspondence* if $G$ and $F$ are quasi-inverses.

The example is a correspondence between a subcateogory of relations and the category of Scott pretopological spaces. This example generalizes the main result in [3].

References


[1] H. Crapo, *Ordered sets: retracts anad connections*, J. of Pure and Applied Algebra, 23 (1982) 13-28.

[2] H. Herrlich and M. Hušek, *Galois connections categorically*, J. of Pure and Applied Algebra, 68 (1990) 165-180.

[3] A. Melton, *Topological Spaces for Cpos*, Lecture Notes in Computer Science, 393 (1989) 302-314.