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or
When "compactness" includes connectedness

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Categeorical Methods in Algebra, Topology and Computer Science

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Theorem (Banaschewski, *On the Weierstrass-Stone approximation theorem*, Fund. Math. 44 (1957), 249–252)

If (X,τ) is a functionally Hausdorff space the Weierstrass-Stone approximation theorem holds for (X,τ) iff (X,τ_w) is compact.

Call (X,τ) a *WS-space*, if (X,τ_w) is compact.

Example (Stephenson 1973)

There exists a *WS*-space X such that X^2 is not a *WS*-space.

For topological spaces (X,τ) and (Y,σ), denote by $\tau\otimes\sigma$ the product topology on $X\times Y$ (resp., $\bigotimes_i \tau_i$ for $\{(X_i,\tau_i) : i\in I\}$)
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CLP-compactness

Definition (A. Šostak, IV Prague TopoSym 1976)

A topological space X is *CLP-compact* if every cover of clopen sets of X has a finite subcover.

compact $\Longrightarrow$ CLP-compact $\Longleftarrow$ connected

A. Sondore, A. Šostak, *On clp-compact and countably clp-compact spaces*, Mathematics, 123–142, Latv. Univ. Zināt. Raksti, 595, Latv. Univ., Riga, 1994.

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This is a weak compactness property: for a space (X, τ) let τ^{CLP} be the topology on X having as a base the τ-clopen sets of X.

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For a space (X,τ), τ^{CLP} is the initial topology of all continuous functions $f\colon (X,\tau)\to D=\{0,1\}\subseteq\mathbb{R}$.
In other words, $(X,\tau)\mapsto(X,\tau^{CLP})$ is the bireflection to the subcategory of zero-dimensional spaces (i.e., $\tau=\tau^{CLP}$ precisely when (X,τ) is zero-dimensional.).

Lemma

The following are equivalent for (X,τ):

(a) (X,τ^{CLP}) *is Hausdorff;*

(b) (X,τ^{CLP}) *is T_0;*

(c) (X,τ) *is totally disconnected .*

For a space X and $x\in X$, let $Q_x(X)$ and $C_x(X)$ be the quasi-component of the point x (i.e., the intersection of all clopen sets containing x) and the connected component of x, respectively. Obviously, $C_x\subseteq Q_x$.
X is totally (hereditarily) disconnected if $Q_x(X)=\{x\}$ (resp. $C_x(X)=\{x\}$) for all $x\in X$.

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The space of quasi-components and CLP-compactness

For a space (X, τ) let rX be the quotient space of all quasi-components of X. Then the quotient map $q_X : X \to rX$ defines a strong epireflection $r : \mathbf{Top} \to \mathbf{TD}$, the subcategory of totally disconnected spaces.

Fact: $r : \mathbf{Top} \to \mathbf{TD}$ preserves and reflects CLP-compactness

This is due to the fact that q_X sends clopen sets to clopen sets.

Definition

Call X strongly CLP-compact, if rX is compact.

$$\begin{array}{ccccc} \text{compact} & \longrightarrow & \text{strongly CLP-compact} & \longrightarrow & \text{CLP-compact} \\ & & \uparrow & & \\ & & \text{connected} & & \end{array}$$

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A subspace X of a topological space Y is *c-dense*, if every connected component of Y meets X.
An embedding $f : (X, \tau) \to (Y, \sigma)$ is CLP-preserving if $f : (X, \tau^{CLP}) \to (Y, \sigma^{CLP})$ is an embedding.

Example

$X \hookrightarrow \beta X$ is CLP-preserving whenever X is Tychonoff.

Theorem (DD, 2007)

Let K be a compact space and let $f : X \to K$ be a continuous map with dense image.

(a) *If X is CLP-compact then $f(X)$ is c-dense in K;*

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Corollary

If X is a pseudocompact space such that $r(\beta X)$ is first countable, then X is CLP-compact.

Follows from the fact that X hits every non-empty G_δ-set of βX (as $r(\beta X)$ is first countable every connected component of βX is a G_δ-set of βX).

Corollary

If $X = \prod_i X_i$ is a Tychonoff pseudocompact space, then X is CLP-compact iff each X_i is CLP-compact.

Lemma (Reduction to Tychonoff spaces)

A totally disconnected space (X, τ) is CLP-compact iff (X, τ_w) is CLP-compact.

Note that $\tau \geq \tau_w \geq \tau^{CLP}$, so

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[A. Šostak, J. Steprāns 1991] If $\{(X_i, \tau_i) : i \in I\}$ are topological spaces such that $\prod_{i \in J}(X_i, \tau_i)$ is CLP-compact for every finite subset of I, does it follow that $\prod_i (X_i, \tau_i)$ is CLP-compact? Does it depend of the size of I?

Some positive results, when
(a) [A. Šostak, J. Steprāns 1991] $|I| < \infty$ and X_i are second countable, or more generally, have $w((X_i, \tau_i) < \mathfrak{p}$ and $w(X_i, {\tau_i}^{CLP}) \leq \omega)$.
(b) [J. Steprāns 2005] $|I| < \infty$ and X_i are sequential.
(c) [DD 2005] Šostak–Steprāns's theorem holds true for arbitrary I.

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Theorem

Let $\{G_i\}_{i\in I}$ be a family of topological groups and $G = \prod_i G_i$.
(a) G is strongly CLP-compact iff each G_i is strongly CLP-compact.
(b) if each G_i is pseudocompact, then G is CLP-compact iff each G_i is CLP-compact.

The proof of (b) uses a theorem of Comfort and Ross about the pseudocompact of products of topological groups.

Theorem

A totally disconnected CLP-compact groups is maximally almost periodic in the sense of J. von Neumann.

The proof uses Ellis' theorem (since the topology τ^{CLP} of a topological group G need not be a group topology [M. Megrelishvili]).

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