Commutative Algebras in Fibonacci Categories

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Abstract.

Let C be a braided monoidal category with braiding $c_{X,Y} : X \otimes Y \to Y \otimes X$. An algebra A in C is said to be *commutative* if $\mu c_{A,A} = \mu$. If C is also equipped with a natural set of isomorphisms $\theta = \{\theta_A : A \to A \,|\, A \in ob\, C\}$, subject to suitable axioms, then we call C *balanced* and the elements of θ are referred to as (ribbon) twists. A commutative algebra A in a balanced category is then called *ribbon* if $\theta_A = 1_A$. If C is also rigid then there is a suitable notion of *separability* for A.

Recall that a tortile (rigid and balanced) monoidal category is said to be fusion when it is semi-simple k-linear together with a k-linear tensor product, finite dimensional hom spaces and a finite number of simple objects (up to isomorphism). We call a fusion category *modular* if it satisfies a certain non-degeneracy (modularity) condition.

A *Fibonacci category* is a modular category with the "fusion rule" $X^2 = 1 + X$. By studying Non-negative Integer Matrix (NIM) representations we show that the Fibonacci category and its tensor powers are completely anisotropic; that is, they do not have any non-trivial separable commutative ribbon algebras.

As an application we deduce that a chiral algebra, with the representation category equivalent to a product of Fibonacci categories, is maximal; that is, it is not a proper subalgebra of another chiral algebra.

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