Towards Noncommutative Gel'fand Duality

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Abstract. Gel'fand-Naimark duality (1943) between the categories of unital commutative C^*-algebras and compact Hausdorff spaces is a key insight of 20th century mathematics, providing an enormously useful bridge between algebra on the one hand and topology and geometry on the other. Many generalisations and related dualitites exist, in logical, localic and constructive forms. Yet, all this is for *commutative* algebras (and distributive lattices of projections, or opens), while in quantum theory and in a large variety of mathematical situations, *noncommutative* algebras play a central role. A good, generally useful notion of spectra of noncommutative algebras is still lacking. Clearly, such spectra will be of considerable interest for physics and Noncommutative Geometry.

I will report on recent progress towards defining spectra of noncommutative operator algebras, mostly for von Neumann algebras. This work comes from the approach using topos theory to reformulate quantum physics (C. Isham, AD), where a presheaf or sheaf topos is assigned to each noncommutative operator algebra, together with a distinguished spectral object. It will be shown that this assignment is functorial, and that the spectral object determines the algebra up to Jordan isomorphisms (J. Harding, AD). Progress on characterising the action of the unitary group of an algebra – relating to Lie group and Lie algebra aspects – is presented. Moreover, recently established connections with Zariski geometries from geometric model theory will be sketched (B. Zilber, AD).

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