

Pseudo-commutativity of lax idempotent 2-monads

Ignacio López Franco

Abstract. The theory of 2-monads provides a framework for the study of categories with extra structure. An algebra for a (finitary) 2-monad on \mathbf{Cat} can be regarded as a category with operations subject to –usually higher-order– equations. Hyland-Power studied the case when these operations commute with each other up to isomorphism, giving rise to the concept of a *pseudo-commutative 2-monad* –a two-dimensional version of the commutative monads studied by A. Kock.– The main example is the 2-monad whose algebras are symmetric strict monoidal categories. We show that every lax idempotent (strong) 2-monad –*i.e.*, with the Kock-Zöberlein property– has a canonical pseudo-commutativity. This provides a large family pseudo-commutative 2-monads, including the 2-monads whose algebras are categories with chosen (co)limits –whose pseudo-commutativity is an incarnation of the fact that “(co)limits commute with (co)limits.”– We discuss internal homs and tensor products arising from the pseudo-commutativities.