Some homotopy methods in the category of graphs

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Abstract. A Quillen model structure on a category $\mathcal{E}$ involves three classes of morphisms (weak equivalences, fibrations, and cofibrations) which obey some weak factorization axioms. Quillen showed that these provide a good framework for describing and working with a universal homotopy functor $\mathcal{E} \to \mathrm{Ho}(\mathcal{E})$ which inverts all the weak equivalences.

Let Gph denote the category of directed irreflexive graphs. After describing graph covering morphisms and some related weak factorization systems, we describe two model structures on Gph. One is based on cycles and the other is based on infinite walks. We calculate the associated homotopy categories and relate them to the study of the zeta series, spectra, and dynamical systems of finite graphs.

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