

# Distributive laws for Lawvere theories

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## *Abstract.*

Lawvere Theories and monads are two related but subtly different ways of handling algebraic theories. Distributive laws give a way of combining two algebraic structures expressed as monads, so one might naturally ask whether something analogous can be done for Lawvere Theories; this question was posed to me by Jean Bénabou in 2009 following my talk on iterated distributive laws for monads. In this talk I will give three ways of doing this. The first uses a generalised notion of “factorisation systems over  $\mathcal{I}$ ” [3], the second uses bimodules as in [2] and the third uses an extension of the free finite-product category 2-monad from **Cat** to **Prof** [1]. In each case Lawvere theories are reformulated as certain monads in a bicategory (as already proposed by Lack and Hyland), and distributive laws between them are then certain distributive laws in that bicategory. I will exhibit comparison functors between **CAT** and each of these bicategories to show that the distributive laws between the Lawvere theories correspond in a suitable way to distributive laws between their associated finitary monads. The three different but equivalent formulations then provide a framework conducive to generalisation, but also an explicit description of the composite theories arising from distributive laws.

## References

- [1] Martin Hyland On distributive laws. Talk at 6th CLP, Oxford 2010.
- [2] Steve Lack Composing props. *Theory and Applications of Categories*, 13(9):147–163, 2004.
- [3] Robert Rosebrugh and R. J. Wood. Distributive laws and factorization. *Journal of Pure and Applied Algebra*, 175:327–353, 2002.