From Differential to Tangential Restriction Categories

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Abstract.

The study of abstract differential structure was initiated by Thomas Ehrhard who encountered an abstract notion of differentiation in his work on the semantics of linear logic. His models are particularly tantalizing as they appear to have a computational content which is, possibly, related to distributed processing. Importantly, for this talk, the work also exposed a purely algebraic/categorical theory of differentiation.

In 2008, Rick Blute, Robert Seely, and I introduced the notion of a Cartesian differential category. Examples of these include the *coKleisli* category of any of the above models of linear logic. However, significantly, Cartesian differential categories also include all the standard (and synthetic models) of differentiation (based on the reals for example). Furthermore, Robert Seely and I were able to show that models of Cartesian differential categories turn out to be precisely coalgebras for the "Faa di Bruno" comonad providing a fairly tight and closed circle for these ideas.

Once one has a purely algebraic and abstract description of differentiation, in hand, the underlying structure of differential geometry comes into reach. However, to support this direction, it is necessary to add partiality (and therefore in some sense "topology") to the differential structure. To this end Geoff Crutwell, Johnathan Gallagher, and I introduced the notion of a differential restriction category. When such has "joins" (which one can always freely add) one may use the manifold construction of Marco Grandis to obtain a category of "smooth" manifolds. The category of smooth manifolds generated from a differential restriction category is almost never itself a differential restriction category. However, its structure can also be axiomatized using tangent bundles and this is what we refer to as "tangential structure".

An central example of this structure arises in algebraic geometry and the purpose of the talk is to introduce this structure motivated by this example.

*Joint work with Geoff Crutwell and Johnathan Gallagher.