Classifying ω-categories of dependent type theories

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Abstract.

Dependent type theories can often profitably be seen as embodied by their *classifying categories*. For extensional theories, this gives a very satisfactory explication of the rules for the logical constructors, as providing various adjoints. For intensional theories, however, this account fails. The connectives are not adjoints, as their uniqueness properties typically only hold up to propositional equality — that is, terms of identity types.

One remedy is to try to see the constructors as providing *weak* logical structure, in some higher-categorical sense. Towards this end, we show that the classifying category of any theory with Martin-Löf identity types underlies a weak ω-category, whose higher cells are terms of identity types. Precisely, we use Batanin/Leinster globular operadic higher categories, and construct the higher-category structure via the co-endomorphism operad of the "type-theoretic globes", a co-globular theory representing the classifying weak ω-category.

This talk is based on the results of my PhD thesis, completed at Carnegie Mellon University under the supervision of Steve Awodey.