Categorical foundations for K-theory

Nicolas Michel

Abstract.

In the different fields where K-theory is applied, the K-theory of an object C is calculated as follows. One first associates to the object C a category $\mathscr{A}_C$ that has a suitable structure (exact, Waldhausen, symmetric monoidal, ...). One then applies to $\mathscr{A}_C$ a "K-theory machine", which provides us with an infinite loop space that is the K-theory $K(C)$ of the object C.

We study the first step of this process. What kind of objects admit a reasonable notion of K-theory? Given these types of objects, what associated structured categories yield meaningful K-theoretic information about the objects? How should the morphisms of these objects interact with this correspondance?

We propose a unified, conceptual framework for a number of imporant examples of objects studied in K-theory. We start with a category $\mathscr{C}$ equipped with a functor to the category of monoids in a (suitably structured) monoidal *fibred* category $\mathscr{E}$. We then associate to the objects of $\mathscr{C}$ subcategories of modules that are *locally trivial* with respect to a chosen class of trivial objects and a Grothendieck-like topology on the category $\mathscr{C}$. We sketch some applications of these new tools for obtaining K-theoretic information. This work is detailed in my PhD thesis*.

*See library.epfl.ch/theses/?nr=4861.