Generalising Connected Components

João J. Xarez

Abstract.

The present work is a generalisation of a previous one by the same author, allowing now to join geometrical examples to the known algebraic and topological examples, in a unified setting. Consider any full reflection $H \vdash I : C \rightarrow M$, with unit $\eta : 1_C \rightarrow HI$, such that $C$ has pullbacks. Suppose there is a functor $U : C \rightarrow S$, and a prefactorisation system $(E, M)$ on $S$, such that $U$ preserves pullbacks, reflects isomorphisms, and $U(\eta_C)$ is in the largest subclass of $E$ which is closed under pullbacks in $S$, $C \in C$. If a certain lemma also holds for a set $T$ of objects in the full subcategory $M$, then it is true that: the reflection $H \vdash I$ is semi-left-exact if and only if its connected components are “connected”; it has stable units if and only if certain pullbacks of connected components are “connected”. The meaning of “connected” is the usual in Galois categorical theory.

References


