Geoffrey Cruttwell *
Mount Allison University

Differential equations in tangent categories

Tangent categories, first defined by Rosický [7], are an abstract setting for differential geometry. Recent work has shown that within their formalism one can define and work with many of the fundamental ideas of differential geometry such as the Lie bracket [3], vector bundles [4], connections [5], and de Rham cohomology [6]. A variety of models for the axioms have also been identified, ranging from examples in ordinary differential geometry to examples in algebraic geometry, synthetic differential geometry, and abelian functor calculus [2, 7, 1]

In this talk, we discuss how to define and work with solutions to ordinary differential equations in tangent categories. This requires several additions to the tangent category axioms. First of all, since solutions to differential equations need not be totally defined, we work in the more general setting of a tangent restriction category (described in [2]) in which maps need only be partially defined. Second, we assume the existence of a special "curve" object which translates vector fields into flows (that is, an object which "solves certain ordinary differential equations"). We will then discuss various consequences of these axioms in this general setting, such as the relationship between the Lie bracket of vector fields and the commutativity of their respective flows.

References:

[1] K. Bauer, B. Johnson, C. Osborne, E. Riehl and A. Tebbe, *Directional derivatives and higher order chain rules for abelian functor calculus*, available at arXiv.org:1610.01930.

[2] J.R.B. Cockett and G.S.H. Cruttwell, *Differential structure, tangent structure, and SDG*, Appl. Categ. Structures 22 (2014), no. 2, 331–417.

[3] J.R.B. Cockett and G.S.H. Cruttwell, *The Jacobi identity for tangent categories*, Cah. Topol. Géom. Differ. Catég., LVI (4) (2015), 301–316.

[4] J.R.B. Cockett and G.S.H. Cruttwell, *Differential bundles and fibrations for tangent categories*, to be published in Cah. Topol. Géom. Differ. Catég.

[5] J.R.B. Cockett and G.S.H. Cruttwell, *Connections in tangent categories*, submitted, available at arXiv.org:1610.08774.

[6] G.S.H. Cruttwell and R. Lucyshyn-Wright, *A simplicial framework for de Rham cohomology in tangent categories*, submitted, available at arXiv.org:1606.09080.

[7] J. Rosický, *Abstract tangent functors*, Diagrammes 12 (1984), JR1–JR11.

*Joint work with Robin Cockett and Rory Lucyshyn-Wright.