Partha Pratim Ghosh

School of Mathematical Sciences,
University of South Africa, Gauteng, South Africa

Internal neighbourhood spaces

The talk generalises the construction of pretopological spaces and pseudotopological spaces to a context where the ground category of sets is replaced with an arbitrary finitely complete category equipped with a proper factorisation system and each lattice of *admissible subobjects* is a complete distributive lattice. It is shown that the categories of *internal weak neighbourhood spaces* and *internal pretopological spaces* are topological over the base category. The category of *internal weak neighbourhood spaces* is shown to be bireflective in the category of *internal pretopological spaces*. In the special case when each lattice of *admissible subobjects* is a pseudocomplemented complete distributive lattice and each change of base a homomorphism of pseudocomplemented complete lattices, the category of *internal pseudotopological spaces* is shown to contain the category of *internal pretopological spaces* bireflectively and is itself topological over the base category. There are *neighbourhood structures* over each object which are similar to the neighbourhoods obtained from a topology on a set. If every change of base is a homomorphism of pseudocomplemented complete lattices then the category of *internal neighbourhood spaces* is topological over the base category and is a bireflective full subcategory of the category of *internal weak neighbourhood spaces*. The special *neighbourhood structures* on an object whose *open subsets* make a topology give rise to *topological structures* on the object. In the special case when each lattice of *admissible subobjects* is a frame and each change of base is a homomorphism of pseudocomplemented complete lattices the category of *internal topological spaces* is isomorphic to the category of *internal neighbourhood spaces* and hence is topological over the base category. Thus, in particular, the classical case for the context of sets and functions is obtained as a special case of the results presented in a more general context in this talk.