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Vector bundles and dependent linear logic in differential geometry

Multicategories provide a categorical semantics for multi-linear maps in linear algebra, and Hermida showed that representable multicategories are equivalent to monoidal categories [1]. Blute-Cockett-Seely introduced systems of linear maps to provide a language for multilinear maps in categories of "smooth" maps, and described when these multilinear maps gave rise to a representable multicategory or a storage comonad [2]. Topological vector bundles - epimorphisms $q : E \to B$ so that for every $b \in B$, $q^{-1}(b)$ is a vector space - give a model of *local* linear structure which is a basic building block in differential geometry.

In this talk, indexed systems of linear maps are developed to model the fibrewise linearity of topological vector bundles. Indexed systems of linear maps gives rise to fiberwise notions of monoidal representability and storage, which in turn gives rise to an indexed monoidal category and the categorical semantics of dependent linear logic [3][4]. This structure is then applied to the differential bundle fibration in a tangent category [5], which was first explored by Cockett and Cruttwell [6], to cleanly express the basic concepts of differential forms and symplectic geometry in a tangent category.

References:

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