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The canonical intensive quality of a pre-cohesive topos

In the context of Lawvere's Axiomatic Cohesion [1], an essential and local geometric morphism $p : \mathcal{E} \to \mathcal{S}$ between toposes is *cohesive* if

i) $p_! : \mathcal{E} \to \mathcal{S}$ preserves finite products.

ii) ("Continuity") for every $E \in \mathcal{E}$ and $S \in \mathcal{S}$ the induced morphism $p_!(E^{(p^*S)}) \to (p_!E)^S$ is an isomorphism.

iii) ("Nullstellensatz") the canonical map $\theta : p_* \to p_!$ is epi.

Without the continuity condition ii), we refer to $p : \mathcal{E} \to \mathcal{S}$ as *pre-cohesive* [3]. For any pre-cohesive $p : \mathcal{E} \to \mathcal{S}$, [1] constructs the associated canonical intensive quality as the full subcategory $\mathcal{L}$ of $\mathcal{E}$ of those objects X for which $\theta_X : p_*X \to p_!X$ is an isomorphism. We call $\mathcal{L}$ the Leibniz category associated to p.

In this talk we will review some of the basic properties of the category $\mathcal{L}$, we will give elementary constructions of the left and right adjoints of the inclusion functor $\mathcal{L} \to \mathcal{E}$, and we will determine sufficient conditions for a pieces preserving geometric morphism [2] $g : \mathcal{F} \to \mathcal{E}$ between two pre-cohesive toposes over $\mathcal{S}$ to restrict to a geometric morphism between the corresponding Leibniz categories.

Furthermore, we will produce a subcanonical site for the Leibniz category determined by the cohesive site over sets of picewise linear functions constructed in [4].

References:

[1] F.W. Lawvere. Axiomatic Cohesion, *Theory and Applications of Categories* 19 (2007) 41–49.

[2] F. Marmolejo and M. Menni. On the relation between continuous and combinatorial, *Journal of Homotopy and Related Structures.* 12 (2017) 379–412.

[3] M. Menni. Sufficient Cohesion over atomic toposes. *Cah. Topol. Gom. Differ. Catég.* 55 (2014) 113-149.

[4] M. Menni. Continuous Cohesion over Sets. *Theory and Applications of Categories,* Vol. 29, No. 20, 2014, pp. 542-568.

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