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Fundamental groupoids for orbifolds

In equivariant topology, we often study G-spaces by regarding them as a diagram of fixed sets $X^H = \{x \in X | hx = x\}$ for various subgroups H. This diagram is indexed over the orbit category O_G, and various topological invariants can be defined by thinking of G-spaces as functors from O_G to Top. One such invariant is tom Dieck's fundamental groupoid, a category defined by taking the fundamental groupoid functor $\Pi : O_G \to Gpoid$ defined by $\Pi(X^H)$, and then combining these using a Grothendieck colimit construction $\Pi_G(X) = \int_{O_G} \Pi(X^-)$ [3, 4].

Orbifolds are locally modelled by group actions, but can be created from charts carrying the action of many different groups, so it is not immediately clear how to create a category to play the role of O_G and organize the fixed point data. Additionally, the orbifold structure can be modeled locally by group actions and globally by groupoids, but this representation is not unique, but only defined up to Morita equivalence. So creating an analogous category for orbifolds presents some challenges.

The category defined by Haefliger [1, 2] incorporates some but not all of the information captured by the tom Dieck construction. It includes some of the internal jumps present in the Grothendieck colimit, but does not include the stratification of the fixed point sets. This category is equivalent to Thurston's deck transformations of the universal cover [4], and to the fundamental group of the classifying space $B\mathcal{G}$ [3]. In this talk, I will expand on the relationship between this category and the tom Dieck category, and discuss ways that the various definitions could lead to an orbifold definition of a tom Dieck fundamental groupoid category.

References:

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[4] Thurston, Orbifolds, `http://library.msri.org/books/gt3m/PDF/13.pdf`.

[5] Scull, L, A van Kampen theorem for equivariant fundamental groupoids, *JPAA* 212 (2009) 2059–2068.

[6] tom Dieck, Transformation Groups, Walter de Gruyter (1987).

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