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Topological theories

In his 2007 paper [1], Hofmann provided a notion of topological theory, involving a **Set**-monad $\mathcal{T}$, a (commutative and unital) quantale $\mathcal{V}$, and a (lax) $\mathcal{T}$-algebra structure on $\mathcal{V}$ that makes the operations of the **Sup**-enriched monoid $\mathcal{V}$ (lax) $\mathcal{T}$-homomorphisms; in addition, the $\mathcal{T}$-structure must satisfy a certain compatibility condition with suprema, which proves to be essential in applications, but does not appear to be well aligned with the other conditions of the notion. Furthermore, in its current form, the notion does not seem to lend itself to generalization, beyond the **Set**-based and quantalic context.

The aim of this talk is to frame Hofmann's notion in the context of a *lax* version of one of the cornerstones of monad theory. In its strict form, given two monads $\mathcal{T}, \mathcal{P}$ on any category $\mathcal{C}$, it describes the interaction and equivalence of the following four algebraic gadgets: distributive laws of $\mathcal{T}$ over $\mathcal{P}$; extensions of $\mathcal{T}$ to the Kleisli category of $\mathcal{P}$; liftings of $\mathcal{P}$ to the Eilenberg-Moore category of $\mathcal{T}$; composite monad structures for $\mathcal{P}T$. In the case at hand, $\mathcal{T}$ may be any **Set**-monad, but is normally assumed to satisfy the Beck-Chevalley condition, and $\mathcal{P}$ is the $\mathcal{V}$-powerset (or presheaf) monad, the Kleisli category of which is the (dual of) the category $\mathcal{V}$-Rel of sets and $\mathcal{V}$-relations.

We show how the various conditions of Hofmann's notion can be made to fit within this framework and to naturally lead to generalizations beyond its current context, as alluded to in part in [2]. Time permitting, we will also discuss examples in the generalized context.

References:

[1] D. Hofmann, Topological theories and closed objects, *Advances in Mathematics* 215 (2007) 789–824.

[2] W. Tholen, Lax distributive laws, I, arXiv:1603.06251.