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A simplicial groupoid for plethysm

Plethysm is a substitution operation in the ring of formal power series in infinitely many variables. It was introduced in the context of unlabelled enumeration in combinatorics (Pólya 1937) and in representation theory of the general linear groups (Littlewood 1944).

In this talk we will give a combinatorial model for this operation in terms of simplicial groupoids. More precisely, we will define the plethystic bialgebra, whose comultiplication is dual to plethystic substitution, and realize it from an explicit Segal space ($T\mathbf{S}$) through the standard constructions of incidence coalgebras and homotopy cardinality of groupoids.

Many combinatorial coalgebras can be obtained using these tools. For instance, we will see that the Faàdi Bruno bialgebra, which is contained in the plethystic bialgebra, can be recovered from the nerve of the category of surjections. The Segal space $T\mathbf{S}$ arises also from the category of surjections by a general categorical construction.