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Mix unitary categories

The standard setting for categorical quantum mechanics is a dagger compact closed category and this framework has been used to explain quantum processes [1] and quantum protocols diagrammatically [2, 4]. However, this framework is fundamentally limited to finite dimensional systems, while quantum physics is not. This talk describes a natural generalization of the standard †-compact closed setting to linearly distributive categories (and $*$-autonomous categories) satisfying the mix law [5] – called **mix categories**. This generalization allows for infinite dimensional systems and – significantly – for many of the techniques developed for compact closed categories to be extended.

A key idea which must be extended from †-categories to linearly distributive categories is the notion of a *unitary isomorphism*. Traditionally this is taken to be an isomorphism f such that $f^{-1} = f^{\dagger}$, however, this relies on having an involution which is stationary. An involution on a mixed category, however, cannot be stationary as it must minimally flip the tensor and par. Thus, in a mixed †-category, it is usual to have a natural isomorphism, called an **involutor**, $\iota : A \to (A^{\dagger})^{\dagger}$ which witnesses that the dagger functor is an involution.

A **unitary object** U, in a mix †-LDC, is an object in the "core" which is equipped with an isomorphism $\varphi_U : U \to U^{\dagger}$ – called the unitary structure of U – satisfying, in particular, that $\varphi_A(\varphi_A^{-1})^{\dagger} = \iota$. The notion of a unitary isomorphism can then be formulated as an isomorphism $u : U \to V$ between unitary objects which is natural with respect to the unitary structure (i.e. $\varphi_U(u^{-1})^{\dagger} = u\varphi_V$). A **mix unitary category** (MUC) is, thus, a mix †-category with class of unitary objects.

We shall provide examples of mix unitary categories and describe some basic constructions, including that of quantum processes, which extend the traditional constructions of categorical quantum mechanics to this setting.

References:

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