Geoff Cruttwell *
Mount Allison University

Affine objects in a tangent category

Affine manifolds are smooth manifolds with a particular choice of charts such that each transition function is affine. In addition to being interesting in their own right, recent work of Jubin has shown that the category of affine manifolds has a wide variety of monads and comonads on it, with many distributive laws relating these monads and comonads [3].

In this talk, we shall see how to define and work with affine objects in a tangent category [2, 4]. A key result of Auslander and Markus shows that an affine structure on a smooth manifold M is equivalent to giving a flat, torsion-free connection on the tangent bundle of M [1]; this we take as the definition of an affine object in a tangent category. We give several alternate characterizations of such connections (some of which appear to be new in the category of smooth manifolds), show how to extend Jubin's results on the existence of monads and comonads to this setting, and find additional distributive laws between these monads and comonads that Jubin did not discover.

References:

[1] Auslander, L. and Markus, L., Holonomy of flat affinely connected manifolds, *Annals of Mathematics* 62 (1955), 139–151.

[2] Cockett, R. and Cruttwell, G., Differential structure, tangent structure, and SDG, *Applied Categorical Structures* 22 (2014) 331–417.

[3] Jubin, B., The tangent functor monad and foliations, arxiv.org:1401.0940 (2014), 50 pages.

[4] Rosický, J., Abstract tangent functors, *Diagrammes* 12 (1984).

*Joint work with Rick Blute and Rory Lucyshyn-Wright.