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Higher commutator conditions for central extensions in Mal'tsev categories

In [1], M.C. Pedicchio introduced a notion of commutator $[R, S]$ for equivalence relations R and S in an exact Mal'tsev category with coequalizers $\mathcal{C}$, which generalizes the classical commutator of normal subgroups in the category of groups and the Lie bracket of ideals in the category of Lie algebras. In the category 2-$\mathbf{Eq}(\mathcal{C})$ of pairs of equivalence relations (on the same object), one can then define the full subcategory $\mathbf{Conn}(\mathcal{C})$ of pairs (R, S) of equivalence relations having trivial commutator : $[R, S] = \Delta_X$. This category is reflective in 2-$\mathbf{Eq}(\mathcal{C})$, with the reflection given by the quotient $X \to \frac{X}{[R,S]}$ by the equivalence relation $[R, S]$.

In this work [2], we define an admissible Galois structure [3] based on this reflection, and characterize central and double central extensions in terms of higher commutator conditions. These results generalize both the ones related to the abelianization functor in exact Mal'tsev categories [4, 5], and the ones corresponding to the reflection from the category of internal reflexive graphs to the one of internal groupoids, which were only known in the varietal case [6].

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