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Projections for Hopf quasigroups

An important result proved by Radford in [5] gives equivalent conditions for the tensor product of two Hopf algebras $B \otimes H$ (equipped with smash product algebra and smash coproduct coalgebra structure) to be a Hopf algebra, and characterizes such objects via bialgebra projections. Later, Majid in [3] interpreted this result in the context of braided categories using the bosonization process. As a consequence, if the antipode of a Hopf algebra H is bijective, we can prove that there is an equivalence between the category of Hopf algebras in the category of left Yetter-Drinfeld modules ${}^H_H\mathcal{YD}$ and the category of projections of H.

An interesting generalization of Hopf algebras are Hopf quasigroups. In a category of vector spaces, this notion was introduced by Klim and Majid in [4]. Hopf quasigroups are not associative but the lack of this property is compensated by some axioms involving the antipode. The concept of Hopf quasigroup includes as example the enveloping algebra of a Mal'tsev algebra as well as the notion of quasigroup algebra of an I.P. loop.

In this talk we introduce the notions of Yetter-Drinfel'd module and strong projection over a Hopf quasigroup H in a symmetric monoidal category. If the antipode of H is an isomorphism, we show that the category of left-Yetter-Drinfel'd modules is braided, and there exists a categorical equivalence between the category of Hopf quasigroups in ${}^H_H\mathcal{YD}$ and the category of strong projections associated to H. Also, we show that for a Hopf quasigroup A and a quasitriangular Hopf quasigroup H, if there exists an invertible skew paring τ, it is possible to obtain a strong Hopf quasigroup projection and, as a consequence, we show that A admits a structure of Hopf quasigroup in ${}^H_H\mathcal{YD}$. This last result provides a way to prove that there exist examples of "truly braided" Hopf quasigroups.

References:

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