J.R.A. Gray *
Stellenbosch University

Algebraic exponentiation and abstract categories of interest

Initiated by the study of simple exactness properties and factorization systems a long time ago, categorical algebra generalizes more and more constructions and results that seemed to be specific to classical algebraic theories, such as theories of groups and of algebras over rings. In this talk we first recall that although the category **Grp** of groups is far from being cartesian closed, it admits two exponent-like constructions. Specifically, given a group G one can form:

(a) the automorphism group $\mathrm{Aut}(G)$;

(b) for any group X the group X^G of maps from G to X.

These two constructions are the essential ingredients which make **Grp** action representable [3] and locally algebraically cartesian closed ([9], [10]), respectively. There are also a few other examples of action representable categories ([3], [4], [1], [2]) and of categories which are locally algebraically cartesian closed ([10], [11], [5]).

The purpose of the talk is three-fold:

- To describe a context in which action representability and locally-algebraic-cartesian closedness become *dual* to each other;
- To explain that the condition requiring the existence of normalizers (in the sense of [12]) together with its dual are the source of many of the categorical properties shared by the categories of (not necessarily unital) rings and various types of algebras (including associative, Boolean and Lie) as well as the all categories of interest in the sense of G. Orzech [13]. In particular we will see that algebraic coherence [8], fiberwise-algebraic-cartesian closedness [5], and action accessibility [7] follow from these two conditions;
- To explain how the theory of complete groups can be generalized to semi-abelian categories, and to explore specific other concrete cases of this generalization.

References:

[1] F. Borceux, D. Bourn, and P. T. Johnstone, Initial normal covers in bi-Heyting toposes, *Archivum mathematicum* 42 (2006) 335–356.

[2] F. Borceux, M. M. Clementino, and A. Montoli, On the representability of actions for topological algebras, *Textos de Matemáticai (Série B)* 46 (2014) 41–56.

[3] F. Borceux, G. Janelidze, and G. M. Kelly, Internal object actions, *Commentationes Mathematicae Universitatis Carolinae* 46 (2005) 235–255.

*Partly based on joint work with D. Bourn, A. S. Cigoli and T. Van der Linden.

[4] F. Borceux, G. Janelidze, and G. M. Kelly, On the representability of actions in a semi-abelian category, *Theory and Applications of Categories* 14 (2005) 244–286.

[5] D. Bourn and J. R. A. Gray, Aspects of algebraic exponentiation, *Bulletin of the Belgian Mathematical Society* 19 (2012) 821–844.

[6] D. Bourn and J. R. A. Gray, Normalizers and split extensions, *Applied Categorical Structures* 23 (2015) 753–776.

[7] D. Bourn and G. Janelidze, Centralizers in action accessible categories, *Cahiers de Topologie et Géométrie Différentielles Catégoriques* 50 (2009) 211-232.

[8] A. S. Cigoli, J. R. A. Gray, and T. Van der Linden, Algebraically coherent categories, *Theory and Applications of Categories* 30 (2015) 1864–1905.

[9] J. R. A. Gray, Algebraic exponentiation and internal homology in general categories, *Ph.D. thesis University of Cape Town* 2010.

[10] J. R. A. Gray, Algebraic exponentiation for categories of lie algebras, *Journal of Pure and Applied Algebra* 216 (2012) 1964–1967.

[11] J. R. A. Gray, Algebraic exponentiation in general categories, *Applied Categorical Structures* 20 (2012) 543–567.

[12] J. R. A. Gray, Normalizers, centralizers and action representability in semi-abelian categories, *Applied Categorical Structures* 22 (2014) 981–1007.

[13] G. Orzech, Obstruction theory in algebraic categories I, II, *Journal of Pure and Applied Algebra* 2 (1972) 287–314 and 315–340.