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Diaconescu's Theorem for 2-toposes of stacks

A special case of Diaconescu's classification theorem (Theorem 4.3 of [1]) shows that flat set-valued functors on a small category are essentially the same as points of the corresponding presheaf topos. Given a Grothendieck topology, it can be shown that the equivalence restricts to one between continuous flat functors and the points of the topos of sheaves (Corollary VII.5.4 of [2], for example).

The goal is to outline an analogous situation for stacks on a Grothendieck site. That is, a flat category-valued pseudo-functor, or opfibration, on a base 1-category is one whose induced tensor product extension along the Yoneda embedding preserves all finite weighted limits. The 2-category of fibrations over the base 1-category classifies flat category-valued functors in the sense that the points of the 2-category of fibrations are biequivalent to flat category-valued pseudo-functors on the base category. A notion of continuity relative to a Grothendieck topology can be introduced for which the foregoing biequivalence restricts to one between the 2-category of flat, continuous category-valued pseudo-functors and the 2-category of "points" of the corresponding 2-category of stacks.

References:

- R. Diaconescu, Change of Base for Toposes with Generators, J. Pure Appl. Alg. 6 (1975) 191–218.
- [2] S. Mac Lane and I. Moerdijk, Sheaves in Geometry and Logic: A First Introduction to Topos Theory, Springer, 1992.