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Effective computations in predicative mathematics

Sets with atoms serve as an alternative to ZFC foundations for mathematics [1], where some infinite, though highly symmetric sets, behave in a finitistic way. Therefore, one can try to carry over classical analysis and algorithms from finite structures to some infinite structures with atoms. Recent results show that this is indeed possible and leads to many practical applications: automata over infinite alphabets [2], model checking [3], constraint satisfaction solving [4], programming languages* [5, 6], to name a few. In this talk we shall take another route to finite analysis of infinite sets, which extends and sheds more light on sets with atoms. Our approach is based on the classical results from topos theory that link first-order theories with pretoposes and then pretoposes, via certain form of completion, with coherent toposes.

The aim of this talk is to set forth foundations for developing algorithms in predicative mathematics to improve effectiveness of methods and tools used in the real-world industrial environment. Possible applications include formal verification of mission and safety critical systems (e.g. systems whose failure or malfunctioning may cause human death). Because there is a strong correspondence between intuitionistic first-order theories and pretoposes, which are well-known foundations for predicative mathematics, this talk can be understood as the study of effective computations in predicative mathematics.

References:

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*A working implementation of a functional programming language capable of processing structures with atoms, is available on the web-site: `https://www.mimuw.edu.pl/%7Eszynwelski/nlambda`.