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A proof of the model-independence of $(\infty, 1)$ -category theory

We prove that four models of $(\infty, 1)$ -categories — quasi-categories, complete Segal spaces, Segal categories, and 1-complicial sets — are equivalent for the purpose of developing ∞ -category theory. To prove this we first introduce the notion of an ∞ -cosmos, an enriched category of fibrant objects in which ∞ -categories live as objects, an example of which is given by each of the four models mentioned above. To each ∞ -cosmos, we construct a “virtual equipment” of ∞ -categories, functors, natural transformations, and modules between ∞ -categories, the last of these being a vehicle to express ∞ -categorical universal properties. We conclude by arguing that the four above mentioned ∞ -cosmoi have biequivalent virtual equipments. The upshot is that ∞ -categorical structures are preserved, reflected, and created by a number of explicit “change-of-model” functors.

REFERENCES:

- [1] Emily Riehl and Dominic Verity, *∞ -Categories for the Working Mathematician*, in preparation (2018), available from www.math.jhu.edu/~eriehl/ICWM.pdf
- [2] Emily Riehl and Dominic Verity, “ ∞ -Category Theory from Scratch,” lecture notes (2015), [arXiv:1608.05314](https://arxiv.org/abs/1608.05314)

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