J. Rosický *
Masaryk University

Forking in accessible categories

Forking is a central notion of model theory, generalizing linear independence in vector spaces and algebraic independence in fields. We develop the theory of forking in arbitrary accessible categories. To do so, we present an axiomatic definition of what we call a stable independence relation and show that this is in fact a purely category-theoretic axiomatization of the properties of model-theoretic forking in a stable first-order theory. The definition lists properties of commutative squares which are declared to be independent and is primarily suitable for accessible categories whose morphisms are monomorphisms, i.e., to categories appearing in model theory.

We show that an accessible category with directed colimits whose morphisms are monomorphisms can have at most one stable independence relation. Its existence has strong consequences, including stability and tameness (in the model-theoretic sense). Any coregular locally presentable category with effective unions has a stable independence relation for regular monomorphisms consisting of pullback squares. Assuming a large cardinal axiom, an accessible category with directed colimits whose morphisms are monomorphisms has a cofinal full subcategory with a stable independence relation if and only if a certain order property fails.

References:

[1] M. Lieberman, J. Rosický and S. Vasey, Forking independence from the categorical point of view, arXiv:1801.09001.

*Joint work with M. Lieberman and S. Vasey.