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Symmetric monoidal categories and Γ*-categories*

A Γ-category is a functor from the category of finite based sets and basepoint preserving functions Γ^{op} to **Cat**. We construct a model category structure on the category of Γ-categories, which is symmetric monoidal closed with respect to the Day convolution product. The fibrant objects in this model category structure are those Γ-categories which are often called *special* Γ-categories. The main objective of this research is to establish a Quillen equivalence between a *natural* model category structure on the category of (small) permutative categories and strict symmetric monoidal functors **Perm** and our model category structure on Γ**Cat**.

The weak equivalences of the natural model category structure are equivalences of underlying categories. In the paper [1], Segal defined a functor from the category of (small) symmetric monoidal categories into Γ**Cat** which can be described as a *nerve functor* for symmetric monoidal categories. The right adjoint $\overline{K}$ of our Quillen equivalence is a *thickening* of Segal's nerve functor. We construct a permutative category $\mathcal{L}$ called *Leinster's category*, having the universal property that each Γ-category extends uniquely to a symmetric monoidal functor along an inclusion functor $\epsilon : \Gamma^{op} \to \mathcal{L}$. The left adjoint P^{Nat} of our Quillen equivalence is a composite functor composed of the symmetric monoidal extension functor indicated above followed by a homtopy colimit functor.

In the paper [2], Mandell had shown that Segal's nerve functor (followed by the ordinary nerve functor) induces an equivalence between a homotopy category of **Perm**, obtained by inverting those strict symmetric monoidal functors which induce a weak homotopy equivalence of simplicial sets upon applying the nerve functor, and a homotopy category of Γ-spaces $\Gamma\mathcal{S}$ obtained by inverting *pre-stable* equivalences which are those maps of Γ-categories which induce a degreewise weak homotopy equivalence of simplicial sets upon applying an E_∞-completion functor. The objective of Mandell's work is to understand the relation between connective spectra and Γ-spaces obtained by applying the Segal's nerve functor to symmetric monoidal categories whereas our objective is to construct a model category of symmetric monoidal categories which is symmetric monoidal closed.

References:

[1] G. Segal, Categories and cohomology theories, *Topology* 13 (1974) 293–312.

[2] M. A. Mandell, An Inverse K-theory functor, *Doc. Math.* 15 (2010) 765–791.