(Co)Homology of crossed modules

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The concept of crossed module has been proved to be significant not only in homotopy theory, but also in many purely algebraic settings, such as group theory, homological algebra, ring theory or algebraic K-theory. Therefore it is quite interesting to study their algebraic structure. My talk concerns with some (co)homological aspects of crossed modules.

We observe that the category of crossed modules is an algebraic category, that is, there is a tripleable underlying functor from the category of crossed modules to the category of sets. This leads to a Barr-Beck cotriple (co)homology for crossed modules which enjoys many desirable properties. These properties are quite similar to those ones for Eilenberg-Mac Lane group (co)homology which, in some cases, turn out to be instances of ours. Thus we establish a sort of Hochschild-Serre 5-term exact sequences for this (co)homology and deduce expressions for the second (co)homology group of a crossed module, generalizing the well known Hopf's and Mac Lane's formulas for group (co)homology. We relate the homology theory of a crossed module to its lower central series and then, applying it to nilpotent crossed modules, we obtain a generalization of some results of groups, due to Stallings and Stammbach. Central extensions of crossed modules are classified by the second cohomology group, while the second homology group for a perfect crossed module is canonically isomorphic to the kernel of its universal central extension. This last result allows us to give a description of universal central extensions in terms of projective presentations, such as it happens in group theory. Finally we show a universal coefficient theorem for this cohomology.

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