Representing polytopes: the Yannakakis theorem

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CMUC - Universidade de Coimbra

15 de Julho de 2014 - *Encontro Nacional da SPM*

Section 1

Definitions and Motivation

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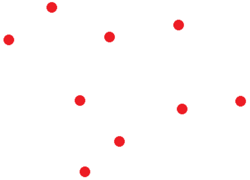

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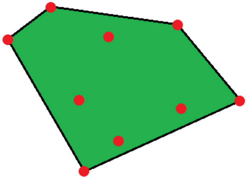

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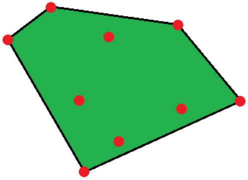

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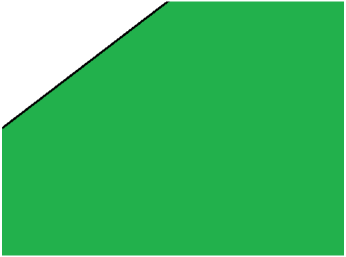

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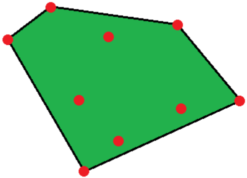

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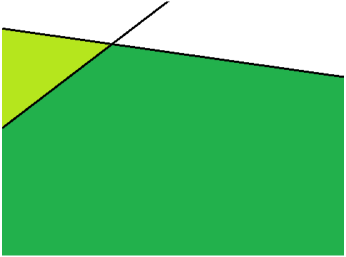

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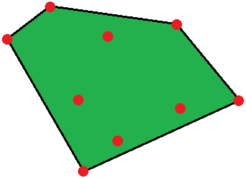

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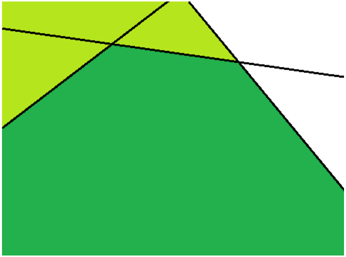

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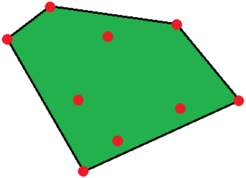

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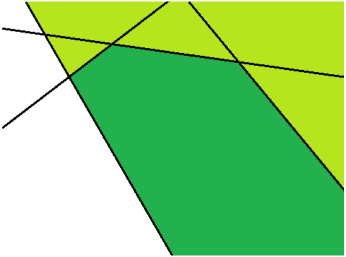

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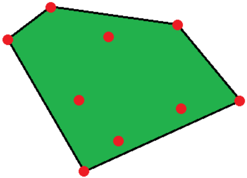

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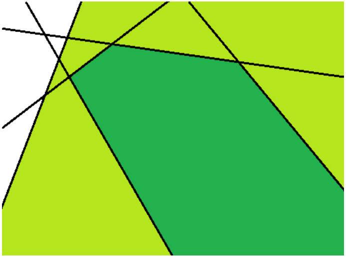

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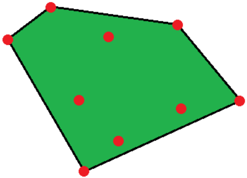

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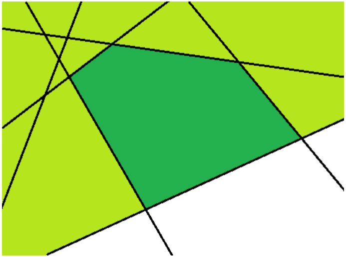

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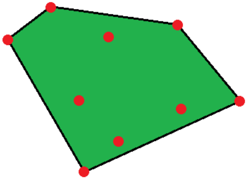

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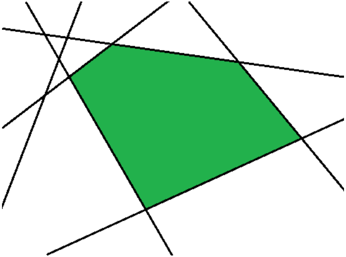

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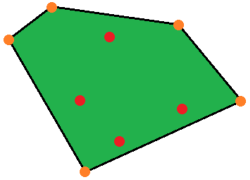

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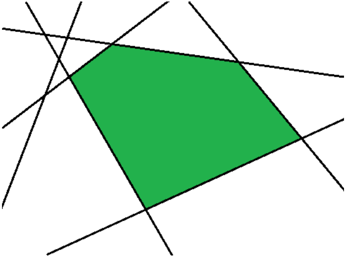

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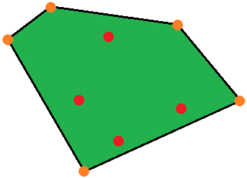

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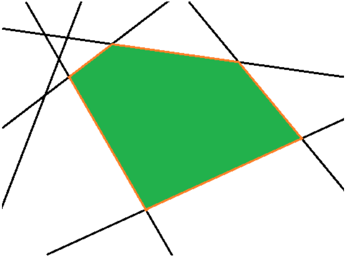

vertices of the polytope $\longleftrightarrow$ minimal set of points

facets of the polytope $\longleftrightarrow$ minimal set of half-spaces

Linear Programming and Polytopes

A linear program is an optimization problem of the type:

$$\begin{array}{ll} \text{maximize} & \langle c, x\rangle \\ \text{subject to} & \begin{array}{l} \langle a_1, x\rangle \leq b_1 \\ \vdots \\ \langle a_m, x\rangle \leq b_m \\ x \in \mathbb{R}^n \end{array} \end{array}$$

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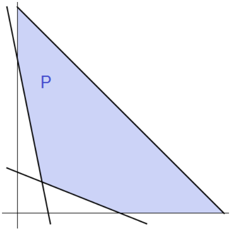

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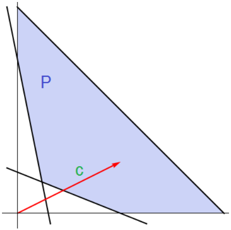

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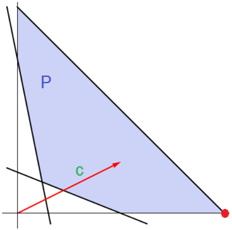

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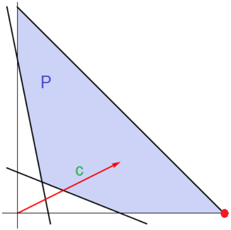

LP is easy: polynomial on the number of facets/vertices

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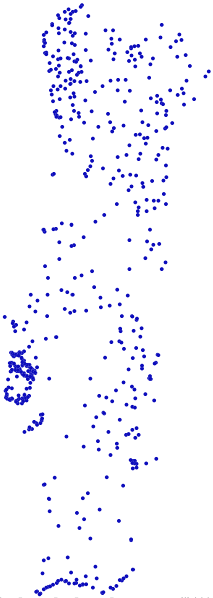

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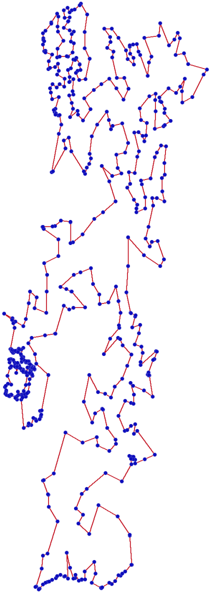

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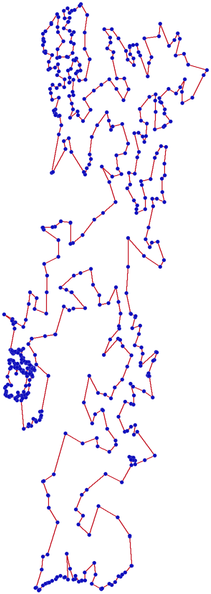

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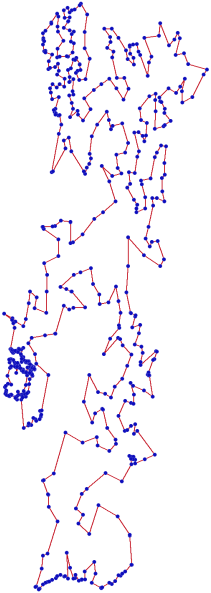

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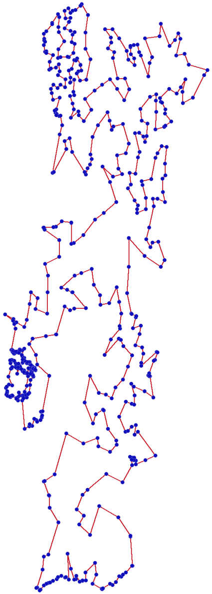
Travelling Salesman Problem Reformulated
Given distances $d_{\{i,j\}}$ from city i to city j solve

$$\begin{array}{ll} \text{minimize} & \langle d, x \rangle \\ \text{subject to} & x \in \text{TSP}(n) \end{array}$$



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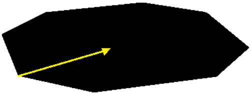

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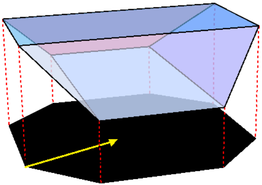

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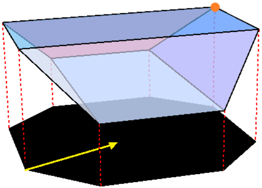

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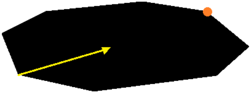

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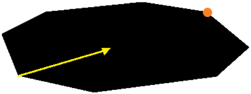

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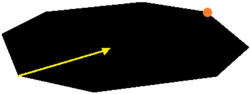

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Regular 2^n-gons can be written as projections of polytopes with $2n$ facets.

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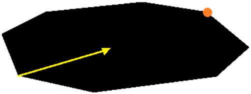

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Parity Polytope

P_n, the convex hull of all $0/1$ vectors with even number of ones, has 2^{n-1} facets and vertices but is the projection of a polytope with $O(n^2)$ facets.

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The main question this poses:

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Attempts at proving $P = NP$ used extensions of the TSP, and one motivation for Yannakakis was to prove them infeasible.

Section 2

Yannakakis Theorem

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Equivalently, it is a collection of vectors $a_1, \cdots, a_m$ and $b_1, \cdots b_n$ in $\mathbb{R}^k_+$ such that $M_{i,j} = \langle a_i, b_j \rangle$.

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Nonnegative Factorizations

Let M be an m by n nonnegative matrix.

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Example:

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

Yannakakis Theorem

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We transform a very hard geometric problem into a very hard algebraic one.

Hexagon

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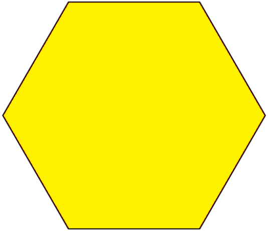

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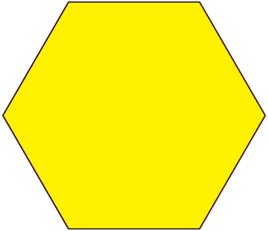

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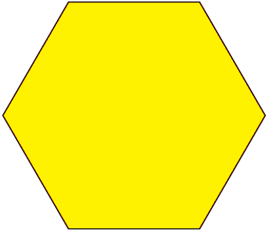

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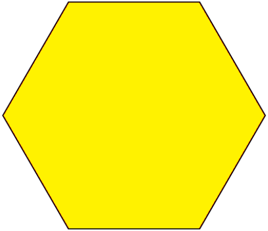

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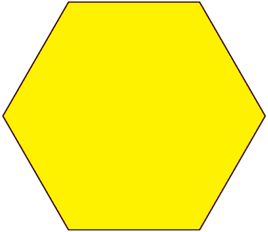

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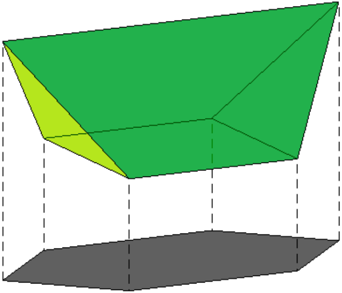

Section 3

Recent results in extension complexity

Travelling Salesman Polytope

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Finally the full result was proven.

Theorem (Fiorini-Massar-Pokutta-Tiwary-Wolf 2012)

xc(TSP(n)) grows exponentially with n.

Matching Problem

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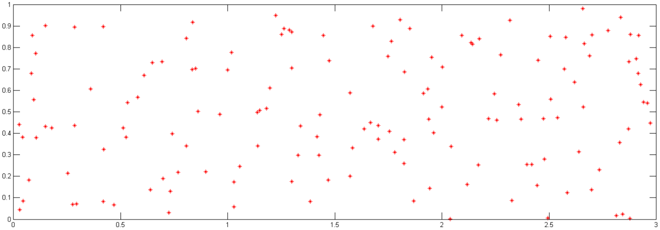

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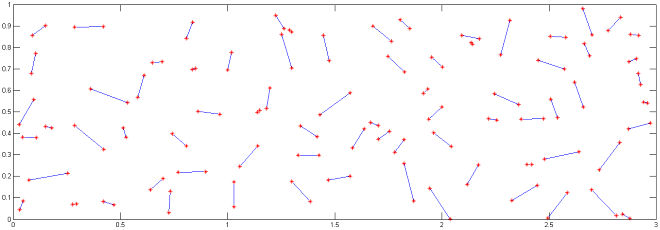

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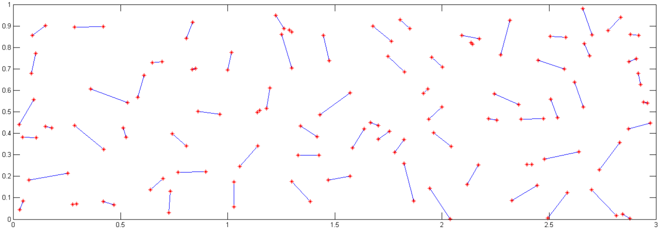

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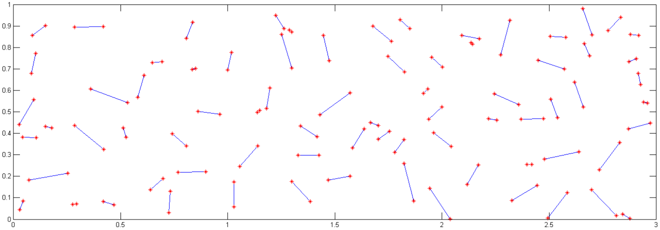

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For any matching M of $2n$ points, let $\chi_M \in \mathbb{R}^{\binom{2n}{2}}$ be defined by $(\chi_M)_{\{i,j\}} = \delta_{\{i,j\} \in C}$. The convex hull of all such points is the matching polytope, MATCH(n).

Matching Problem (continued)

Matching Problem Reformulated

Given distances $d_{\{i,j\}}$ from point i to point j solve

$$\begin{array}{ll} \text{minimize} & \langle d, x \rangle \\ \text{subject to} & x \in \text{MATCH}(n) \end{array}$$

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Matching Problem Reformulated

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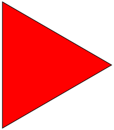

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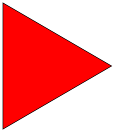

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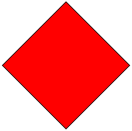

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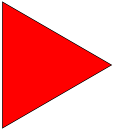

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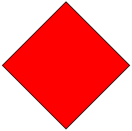

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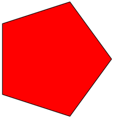

$\text{xc}(P_5) = 5$

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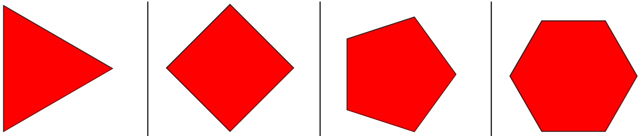

$\mathrm{xc}(P_3) = 3$ | $\mathrm{xc}(P_4) = 4$ | $\mathrm{xc}(P_5) = 5$ | $\mathrm{xc}(P_6) = 5$

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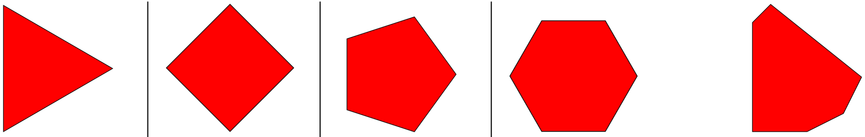

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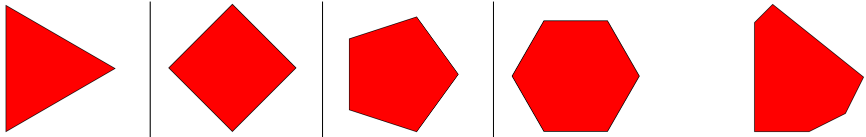

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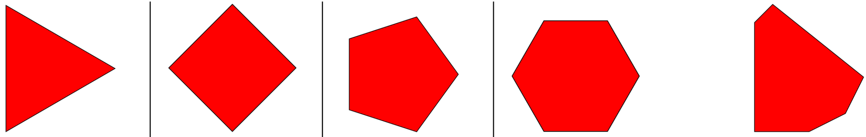

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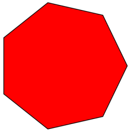

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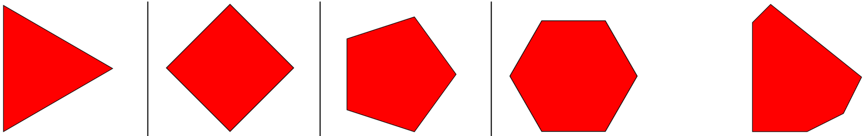

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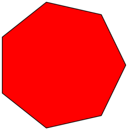

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Corollary

All n-gons have extension complexity at most $\lceil 6n/7 \rceil$.

Polygons (continued)

Lemma

The extension complexity of an n-gon is at least $\log_2(n)$.

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Theorem (Fiorini - Rothvoss - Tiwary 2011)

The extension complexity of a generic n-gon is at least $\sqrt{2n}$.

Section 4

Semidefinite extension complexity

Semidefinite programming

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A semidefinite program (SDP) is an optimization problem of the type:

$$\begin{array}{ll} \text{maximize} & \langle c, x \rangle \\ \text{subject to} & \begin{array}{l} \sum_{i=1}^m A_i x_i \succeq 0 \\ x \in \mathbb{R}^n \end{array} \end{array}$$

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- SDP is efficiently solvable.

Semidefinite extension complexity

A semidefinite representation of size k of a polytope P is a description

$$P = \left\{ x \in \mathbb{R}^n \;\middle|\; \exists y \text{ s.t. } A_0 + \sum A_i x_i + \sum B_i y_i \succeq 0 \right\}$$

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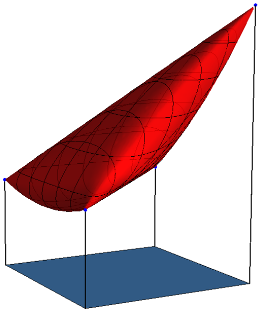

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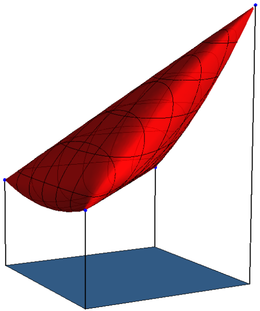

The smallest k for which such a representation exists is the semidefinite extension complexity of P, $\mathrm{xc}_{\mathrm{psd}}(P)$.

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$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1/2 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{matrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \end{matrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Semidefinite Factorizations

Let M be a m by n nonnegative matrix. A PSD_k-factorization of M is a set of $k \times k$ positive semidefinite matrices $A_1, \cdots, A_m$ and $B_1, \cdots B_n$ such that $M_{i,j} = \langle A_i, B_j \rangle$.

$$\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1/2 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

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The smallest k for which such factorization exists is the positive semidefinite rank of M, $\text{rank}_{\text{psd}}(M)$.

Generalized Yannakakis Theorem

Theorem (G-Parrilo-Thomas 2013)

Let P be any polytope and S its slack matrix.

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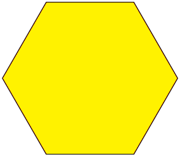
In fact this theorem is more general than just polytopes and semidefinite representations.

The Hexagon

Consider again the regular hexagon.



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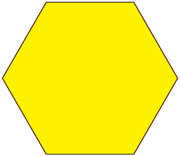

Its 6×6 slack matrix.

$$\begin{bmatrix} 0 & 0 & 2 & 4 & 4 & 2 \\ 2 & 0 & 0 & 2 & 4 & 4 \\ 4 & 2 & 0 & 0 & 2 & 4 \\ 4 & 4 & 2 & 0 & 0 & 2 \\ 2 & 4 & 4 & 2 & 0 & 0 \\ 0 & 2 & 4 & 4 & 2 & 0 \end{bmatrix}$$

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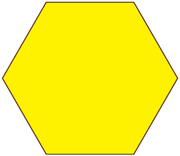

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$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix},$$

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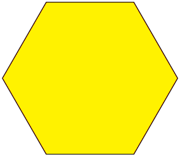

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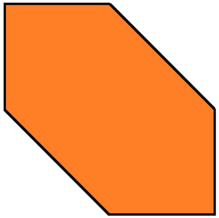
Consider the affinely equivalent hexagon H with vertices $(\pm 1, 0), (0, \pm 1), (1, -1)$ and $(-1, 1)$.



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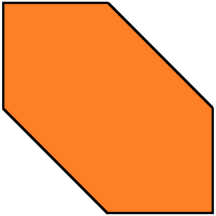

$$H = \left\{ (x_1, x_2) : \begin{bmatrix} 1 & x_1 & x_2 & x_1 + x_2 \\ x_1 & 1 & y_1 & y_2 \\ x_2 & y_1 & 1 & y_3 \\ x_1 + x_2 & y_2 & y_3 & 1 \end{bmatrix} \succeq 0 \right\}$$

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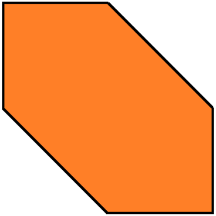

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In fact:

Theorem (G-Robinson-Thomas *2013+*)

All hexagons have semidefinite extension complexity 4.

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A popular candidate for the last question is the polytope $\mathrm{STAB}(G)$ of a perfect graph, where $\mathrm{STAB}(G)$ is just the LP formulation of the max stable set problem.

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- Useful upper/lower bounds for positive semidefinite rank.
- Explore connection to statistics and quantum computing.
- Understand the role of symmetry.

Conclusion

To learn more about this work:

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Positive semidefinite rank.
Coming soon...

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