ON THE ALGEBRAIC NATURE OF THE DUAL CATEGORY OF VIETORIS COALGEBRAS

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The motivation for this talk stems from two very different sources. Firstly, it is known since the 1970's that the dual of the category of compact Hausdorff spaces and continuous maps is a variety; not finitary, but bounded by $\aleph_1$. A description of its algebra operations was first presented in [2], and a complete description of its algebraic theory was obtained only recently in [4]. Our second source of inspiration is the theory of coalgebras. In [3], the authors argue that the category of Boolean spaces and continuous maps "is an interesting base category" for the study of coalgebras, and consider in particular coalgebras for the Vietoris functor. A similar study can be found in [1] with Priestley spaces instead of Boolean spaces. Arguably, these categories are very suitable in this context because they are duals of finitary varieties, which facilitates the development of a useful duality theory for categories of coalgebras.

In this talk we make the case that similar observations apply to the larger category $\mathsf{PosComp}$ of partially ordered compact spaces and monotone continuous maps. We show that $\mathsf{PosComp}^{\mathrm{op}}$ is a $\aleph_1$-ary quasivariety, give a partial description of its algebraic theory which is sufficient to identify also the dual of the category of Vietoris coalgebras as a $\aleph_1$-ary quasivariety, and characterise the $\aleph_1$-copresentable objects of $\mathsf{PosComp}$.

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References

[1] M. M. Bonsangue, A. Kurz, and I. M. Rewitzky, *Coalgebraic representations of distributive lattices with operators*, Topology and its Applications, 154 (2007), pp. 778–791.

[2] J. R. Isbell, *Generating the algebraic theory of $C(X)$*, Algebra Universalis, 15 (1982), pp. 153–155.

[3] C. Kupke, A. Kurz, and Y. Venema, *Stone coalgebras*, Theoretical Computer Science, 327 (2004), pp. 109–134.

[4] V. Marra and L. Reggio, *Stone duality above dimension zero: Axiomatising the algebraic theory of $C(X)$*, Advances in Mathematics, 307 (2017), pp. 253–287.

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