A general limit lifting theorem for 2-dimensional monad theory

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An important question for monad theory is the possibility of lifting limits along the forgetful functor of the categories of algebras. The article I will present (arXiv:1702.03303) deals with this subject within the theory of 2-categories. For strict morphisms of algebras, it is well known that all limits lift. However, as it is usually the case, the relevant notions for its applications are the weaker pseudo and lax notions, and for these notions it is no longer the case that all limits lift. There are in the literature lifting results for the 2-categories of pseudo and lax morphisms of algebras, with the two cases treated separately.

I will present a definition of weak morphism of T-algebras, for a 2-monad T, which includes the notions of lax, pseudo and strict morphisms as particular cases. I will also give a general notion of weak limit, and define what it means for such a limit to be compatible with another family of 2-cells. These concepts allow to state and prove a limit lifting theorem which unifies and generalizes the results that were known for the particular cases of lax, pseudo and strict morphisms.