

## BOOKS

Stampacchia, he introduced the concept of a variational inequality, which proved to be very useful in many applications.

The second volume of the series with subtitles "Control" and "Homogenization", is introduced by A. Bensoussan and contains results published from the end of the sixties up to the end of the nineties. J.-L. Lions' interest in optimal control theory started soon after the pioneering books of L. S. Pontryagin and R. Bellman, which appeared in 1957. Already 11 years later, Lions' book "Control optimal des systèmes ..." was published, and it became soon a standard reference book. It was well known that the problem of optimal control for distributed systems can lead to problems with free boundary of Stefan type. J.-L. Lions used variational inequalities to make this relation straightforward. In the seventies, he generalized the notion of variational inequalities so that it was well suited to describe the problems of optimal stopping time and together with A. Bensoussan, they studied the stochastic control and variational inequalities and impulse control and quasi-variational inequalities. Later he returned to optimal control theory from a new point of view - namely the possibility of using control theory for stabilization of unstable or ill posed problems. New aspects of control theory appear also in J.-L. Lions later papers, especially his famous HUM - Hilbert uniqueness method. Together with R. Glowinsky, he introduced numerical methods suitable for solving these questions. In the second half of the seventies, J.-L. Lions was interested in the homogenisation theory for a large scale of equations with periodically oscillating coefficients. The method he used was far-reaching and applicable also to problems with an oscillating boundary or problems posed on perforated domains. Thus it made it possible in fluid mechanics to deduce Darcy's equation from Stokes' system, or to study the homogenisation of Bingham fluids. At the same time it allowed the study of mechanical properties of composite materials.

The third volume, with an introduction by P. G. Ciarlet, is devoted to numerical analysis and applications of PDEs in a wide scale of problems - the mechanics of fluids and solid bodies, Bingham fluids, viscoelastic or plastic materials, problems with friction, and plates described by linear as well as nonlinear elasticity theory. Since 1990, J.-L. Lions was attracted by very complex and complicated systems of PDEs, i.e., by climatology models. Even in this exceptional case, he, together with R. Temam and S. Wand, proved existence and uniqueness of solutions, their asymptotic behaviour and ways to their numerical solution. These two papers and more than 20 others dealing with highly interesting problems are contained in Volume III. The papers in the collection are, as well as all works by J.-L. Lions, written in a very clear and concise form and they are indispensable for researchers in PDEs and in numerical analysis. (jsta)

**Y. Lu: Hyperbolic Conservation Laws and the Compensated Compactness Method, Monographs and Surveys in Pure and Applied Mathematics 128, Chapman & Hall/CRC, Boca Raton, 2003, 241 pp., \$84,95, ISBN 1-58488-238-7**

The book is devoted to the theory of the compensated compactness method, which is a principal tool for studying properties of systems of hyperbolic conservation laws. Quasilinear systems of hyperbolic conservation laws in one space dimension are considered. This setting (systems in one space dimension) is more or less the only case for which the existence and uniqueness results can be obtained also in a classical way, e.g. by the method of wave-front tracking. In this book a different approach is used, namely that of compensated compactness. The author introduces basic elements of the theory of compensated compactness, based on results of Tartar and Murat from the 80's. The notion of a Young measure is introduced and discussed. After these preliminaries, the author studies the Cauchy problem for a scalar equation with  $L^\infty$  and  $L^p$  data. It is to be noted that the simplified proof of the existence of the solution presented here does not need to use a concept of the Young measure. In the system case, the author works with all the usual concepts, such as the strict hyperbolicity, genuine non-linearity, linear degeneracy, Riemann invariants, entropy-entropy flux pair and theory of invariant regions to obtain uniform  $L^\infty$ -infinity estimates, symmetric and symmetrizable systems. The author then studies different important systems of hyperbolic equations, namely the system of Le Roux type, the system of the polytropic gas dynamic, Euler equations of one-dimensional compressible fluid flow, systems of elasticity and some appli-

cations of the compensated compactness to relaxation problems. The book is carefully written and will be appreciated both by PhD students and experts in the field as one of a few books gathering the knowledge until recently dispersed among research papers. (mro)

**G. Mislin, A. Valette: Proper Group Actions and the Baum-Connes Conjecture, Advanced Courses in Mathematics CRM Barcelona, Birkhäuser, Basel, 2003, 131 pp., €28, ISBN 3-7643-0408-1**

The book has two parts. The first contribution, by G. Mislin, contains a discussion of the equivariant  $K$ -homology  $KG^*(EG)$  of the classifying space  $EG$  for proper actions of a group  $G$ . The Baum-Connes conjecture states that the  $K$  theory of the reduced  $C^*$  algebra of a group  $G$  can be computed by the equivariant  $K$ -homology  $KG^*(EG)$ . The tools used in the exposition contain the Bredon homology for infinite groups. Relations of the Baum-Connes conjecture to many other famous conjectures in topology are described in the Appendix. The second part (written by A. Valette with Appendix by D. Kucerovsky) contains a discussion of the Baum-Connes conjecture for a countable discrete group  $\Gamma$ . Suitable index maps provide the link between both sides of the conjecture. The second part of the book contains a careful discussion of these maps. Both lecture notes clearly cover the area around a beautiful, interdisciplinary field and could be very useful to anybody interested in the subject. (vs)

**V. Müller: Spectral Theory of Linear Operators and Spectral Systems in Banach Algebras, Operator Theory Advances and Applications, vol. 139, Birkhäuser, Basel, 2003, 381 pp., €158, ISBN 3-7643-6912-4**

The monograph is written as an attempt to organize a huge amount of material on spectral theory, most of which was until now available only in research papers. The aim is to present a survey of results concerning various types of spectra in a unified, axiomatic way. The book is organized in to five chapters. At the beginning, the author presents spectral theory in Banach algebras, which forms a natural frame for spectral theory of operators. The second chapter is devoted to applications to operators. Of particular interest are regular functions: operator-valued functions whose ranges behave continuously. A suitable choice of a regular function gives rise to the important class of Kato operators and the corresponding Kato spectrum. The third chapter gives a survey of results concerning various types of essential spectra, Fredholm and Browder operators, etc. The fourth chapter contains an elementary presentation of the Taylor spectrum, which is by many experts considered to be the proper generalization of the ordinary spectrum of a single operator. The most important property of the Taylor spectrum is existence of a functional calculus for functions analytic on a neighbourhood of the Taylor spectrum. The last chapter is concentrated on the study of orbits and weak orbits of operators, which are notions closely related to the invariant subspace problem. All results are presented in an elementary way. Only a basic knowledge of functional analysis, topology and complex analysis is assumed. Moreover, basic notions and results from Banach spaces, analytic and smooth vector-valued functions and semi-continuous set-valued functions are given in the Appendix. The monograph should appeal both to students and to experts in the field. It can also serve as a reference book. (jsp)

**M. C. Pedicchio, W. Tholen, Eds.: Categorical Foundations: Special Topics in Order, Topology, Algebra, and Sheaf Theory, Encyclopaedia of Mathematics and Its Applications 97, Cambridge University Press, Cambridge, 2004, 440 pp., \$90, ISBN 0-521-83414-7**

The book is a result of a collaborative research project of mathematicians from four European and three Canadian universities. During the years 1998 - 2001, small teams were formed to work on a variety of themes of current interest and to develop the categorical approach to them. This book presents the results of their work. The book contains 8 chapters devoted in turn to ordered set and adjunction (by R. J. Wood), locales (by J. Picado, A. Pultr and A. Tozzi), general topology (by M. M. Clementino, E. Giuli and W. Tholen), regular, protomodular and Abelian categories (by D. Boum and M. Gran), monads (by M. C. Pedicchio and F. Rovatti), sheaf theory (by C. Centazzo and E. M. Vitale) and to effective descent morphisms (by G. Janelidze, M. Sobral and W. Tholen). Every chapter is self-contained with its own list of references. The book is

very comprehensive and presents a lot of material on each of the themes. Moreover, it offers various ways of how to study "spaces" or "algebras", selecting and accentuating some of their features. The knowledge required for the reading of the book varies between the chapters, but only a modest knowledge of category theory is supposed at the beginning. The authors of each chapter develop the necessary categorical techniques themselves. The book will be very useful for graduate students and teachers, and inspiring for the researchers interested in the discussed topics as well as in category theory itself. (vtr)

**G. Pisier: Introduction to Operator Space Theory, London Mathematical Society Lecture Note Series 294, Cambridge University Press, Cambridge, 2003, 475 pp., £39,95, ISBN 0-521-81165-1**

The monograph is devoted to the study of operator space theory. The book has three parts. The first part contains a basic exposition of the theory and various illustrative examples. An operator space is just a (usually complex) Banach space  $X$ , equipped with a given embedding into the space  $B(H)$  of all bounded linear operators on a Hilbert space  $H$ . Natural morphisms in this category are completely bounded maps (a linear map is completely bounded if the naturally induced mappings between respective spaces of matrices have uniformly bounded norms). In view of this, we can have many operator space structures on a given Banach space. One of the basic tools for the theory is the Ruan theorem, which shows the correspondence between operator space structures on  $X$  and some kind of norms on the tensor product of  $X$  with the space  $K(L_2)$  of compact operators on  $L_2$ . Basic operations on Banach spaces (dual space, quotient space, direct sum, complex interpolation, some tensor products, etc.) can be defined in the category of operator spaces. Some of these definitions are elementary, some make use of the Ruan theorem. It is also worth mentioning the existence (and uniqueness) of the "operator Hilbert space" - a canonical operator space structure on the Hilbert space. The second part is devoted to  $C^*$ -algebras, which form a subclass of operator spaces (notice that the structure of a  $C^*$ -algebra on a Banach space induces a unique operator space structure). The main themes in this part are  $C^*$ -tensor products and various classes of  $C^*$ -algebras and von Neumann algebras. Some properties of  $C^*$ -algebras are extended to general operator spaces, and local theory of operator spaces is investigated. The third part deals with non-self-adjoint operator algebras, their tensor products and free products. The theory of operator spaces is also used to reformulate some classical similarity problems. (okal)

**A. Polishchuk: Abelian Varieties, Theta Functions and the Fourier Transform, Cambridge Tracts in Mathematics 153, Cambridge University Press, Cambridge, 2003, 292 pp., £47,50, ISBN 0-521-80804-9**

The book gives a modern introduction to theory of Abelian varieties and their theta functions. The text is based on lectures by the author delivered at Harvard University (1998) and Boston University (2001) and it provides an up-to-date introduction to the subject, oriented to the general mathematical community. One of the main goals is to give the first introduction to algebraic theory of Abelian varieties and theta functions, employing Mukai's approach to the Fourier transform in the context of Abelian varieties. This approach is also supported by recently discovered links to the mirror symmetry problem in algebraic geometry and quantum field theory. The exposition of material presented in the book was influenced by the category approach to problems under consideration. The book is divided into three main parts, then each of them into seven or eight chapters. Part I (Analytic Theory) discusses classical and recent aspects of transcendental theory of Abelian varieties. Part II (Algebraic Theory) is devoted to general Abelian varieties over an algebraically closed field of arbitrary characteristic. Part III (Jacobians) contains theory of Jacobian varieties of smooth irreducible projective curves over arbitrary fields. The chapters' text contains many exercises complementing the material covered. The bibliography is up-to-date and comprehensive, consisting of 138 titles. The book is primarily intended for anybody interested in modern algebraic geometry and mathematical physics, with a good background not only in complex and differential geometry, classical Fourier analysis, or representation theory, but also in modern algebraic geometry and categorical algebra. The book is written by a leading expert in the field and it will certainly be a valuable enhancement to the existing literature. (špor)