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Extended real functions in pointfree topology. (English summary)
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Given a frame L, let $\mathcal{S}(L)$ be the dual of the co-frame of all sublocales of L. The frame of extended reals $\mathfrak{L}(\overline{\mathbb{R}})$ is the one having generators of the form $(r,-)$ and $(-,r)$, where $r \in \mathbb{Q}$, subject to the following relations: (1) $(r,-) \wedge (-,s) = 0$ if $r \geq s$, (2) $(r,-) \vee (-,s) = 1$ if $r < s$, (3) $(r,-) = \bigvee_{s>r}(s,-)$ for all $r \in \mathbb{Q}$, and (4) $(-,r) = \bigvee_{s<r}(-,s)$ for all $r \in \mathbb{Q}$. Then $\overline{F}(L)$ is the set of all frame homomorphisms $f\colon \mathfrak{L}(\overline{\mathbb{R}}) \to \mathcal{S}(L)$ which are called extended real functions on L. Further, an $f \in \overline{C}(L) \subseteq \overline{F}(L)$ is called continuous if $f(r,-)$ and $f(-,r)$ are closed sublocales for all $r \in \mathbb{Q}$. The paper deals with the algebraic aspects of $\mathfrak{L}(\overline{\mathbb{R}})$ and their extended function algebras by using scales to generate the extended real functions on L. More specific results concern a sublattice $D(L)$ consisting of those $f \in \overline{C}(L)$ for which $\bigvee_{r<s} f(r,s)$ is dense. Even if, in general, $D(L)$ is not a group or a ring under the operations in $\overline{C}(L)$, there is a lattice-ordered ring isomorphism between $D(L)$ and the ring $C(\mathfrak{B}L)$ of all continuous functions on the Booleanization $\mathfrak{B}L$ of L provided L is extremally disconnected.

Reviewed by *Tomasz Kubiak*

References

1. R.N. Ball, J. Walters-Wayland, C- and C^*-quotients in pointfree topology, Dissertationes Mathematicae (Rozprawy Mat.) 412 (2002) 62. MR1952051 (2004a:06007)
2. B. Banaschewski, The real numbers in Pointfree Topology, in: Textos de Matemática, vol. 12, University of Coimbra, 1997. MR1621835 (99c:54017)
3. B. Banaschewski, Uniform completion in Pointfree Topology, in: S.E. Rodabaugh, E.P. Klement (Eds.), Topological and Algebraic Structures in Fuzzy Sets, in: Trends in Logic, vol. 20, Kluwer Academic Publishers, Boston, Dordrecht, London, 2003, pp. 19–56. MR2046735
4. B. Banaschewski, On the function ring functor in Pointfree Topology, Appl. Categ. Structures 13 (2005) 305–328. MR2175953 (2006i:06026)
5. B. Banaschewski, A new aspect of the cozero lattice in pointfree topology, Topology Appl. 156 (2009) 2028–2038. MR2532132 (2010j:54022)
6. B. Banaschewski, S.S. Hong, Completeness properties of function rings in pointfree topology, Comment. Math. Univ. Carolinae 44 (2003) 245–259. MR2026162 (2004i:54016)
7. B. Banaschewski, A. Pultr, Booleanization, Cahiers Topologie Géom, Différentielle Catég. 37 (1996) 41–60. MR1383446 (97b:06006)
8. T. Dube, M. Matlabyane, Notes concerning characterizations of quasi-F frames, Quaest. Math. 32 (2009) 551–557. MR2598206 (2011b:06013)
9. L. Gillman, M. Jerison, Rings of Continuous Functions, D. Van Nostrand, 1960. MR0116199

(22 #6994)
10. J. Gutiérrez García, T. Kubiak, J. Picado, Localic real functions: a general setting, J. Pure Appl. Algebra 213 (2009) 1064–1074. MR2498797 (2010a:06020)
11. J. Gutiérrez García, J. Picado, On the algebraic representation of semicontinuity, J. Pure Appl. Algebra 210 (2007) 299–306. MR2319999 (2008b:54025)
12. J. Gutiérrez García, J. Picado, Rings of real functions in pointfree topology, Topology Appl. 158 (2011) 2264–2278. MR2838376 (2012i:06020)
13. P.T. Johnstone, Stone Spaces, Cambridge University Press, 1982. MR0698074 (85f:54002)
14. Y.-M. Li, G.-J. Wang, Localic Katětov-Tong insertion theorem and localic Tietze extension theorem, Comment. Math. Univ. Carolinae 38 (1997) 801–814. MR1603726 (99b:54024)
15. J. Picado, A. Pultr, Locales mostly treated in a covariant way, in: Textos de Matemática, vol. 41, University of Coimbra, 2008. MR2459570 (2010d:06012)
16. S. Vickers, Toposes pour les vraiment nuls, in: Jourdan Edalat, McCusker (Eds.), Theory and Formal Methods 1996, Third Imperial College Department of Computing Workshop on Theory and Formal Methods, Imperial College Press, London, 1996, pp. 1–12.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.