

MR2979647 (Review) 06D22 (06D15 54E05 54E15 54E55)

Picado, Jorge (P-CMBR-CM); Pultr, Aleš (CZ-KARLMP-AM)

On strong inclusions and asymmetric proximities in frames. (English summary)

Order **29** (2012), *no. 3*, 513–531.1572-9273

Here a strong inclusion is a specific kind of subrelation of the order of a lattice with pseudocomplements. It can be used in the case of the lattice of open sets in topology to define a proximity. It then also yields a pointfree extension of this concept. A modification of a strong inclusion for biframes then further provides a pointfree model for the notion of a quasi-proximity.

As a sequel of their work about cover quasi-uniformities in frames, in this paper the authors establish that, when symmetry is dropped, biframes can essentially be avoided. Indeed, the concept of a strong inclusion can be asymmetrically modified to work directly on frames, without the prior assumption of an underlying biframe structure. The category of quasi-proximal frames obtained by the authors is proved to be concretely isomorphic with the one based on biframes, and it is related to the category of quasi-uniform frames in the expected way. Note that in the classical asymmetric theory bitopologies also only appear a posteriori.

Reviewed by *Hans-Peter A. Küenzi*

References

1. Banaschewski, B.: Compactification of frames. *Math. Nachr.* **149**, 105–116 (1990) [MR1124796 \(92j:54034\)](#)
2. Banaschewski, B., Brummer, G.C.L., Hardie, K.A.: Biframes and bispaces. *Quaest. Math.* **6**, 13–25 (1983) [MR0700237 \(84h:06012\)](#)
3. Doitchinov, D.: Some reflections on quasi-uniform frames. In: *Topology with Applications* (Szekszàrd, Hungary, 1993), vol. 4, pp. 151–158. Bolyai Society, Math. Studies (1995) [MR1374802 \(97e:54025\)](#)
4. Dowker, C.H.: Mappings of proximity structures. In: *General Topology and its Relation to Modern Analysis and Algebra* (Proc. Sympos., Prague, 1961), pp. 139–141. Academic Press, New York; Publ. House, Czech. Acad. Sci., Prague (1962)
5. Fletcher, P., Hunsaker, W.: Totally bounded uniformities for frames. *Topol. Proc.* **17**, 59–69 (1992) [MR1255795 \(95c:54044\)](#)
6. Gantner, T.E., Steinlage, R.C.: Characterizations of quasi-uniformities. *J. London Math. Soc.* **5** (2), 48–52 (1972) [MR0380741 \(52 #1638\)](#)
7. Frith, J.: Structured frames. Ph.D. thesis, University of Cape Town (1987)
8. Frith, J., Schauerte, A.: The Samuel compactification for quasi-uniform biframes. *Topol. Appl.* **156**, 2116–2122 (2009) [MR2532140 \(2010d:06011\)](#)
9. Hunsaker, W., Lindgren, W.: Construction of quasi-uniformities. *Math. Ann.* **188**, 39–42 (1970) [MR0266149 \(42 #1057\)](#)
10. Hunsaker, W., Picado, J.: A note on total boundedness. *Acta Math. Hung.* **88**, 25–34 (2000)

[MR1780509 \(2001h:54044\)](#)

11. Kelly, J.C.: Bitopological spaces. *Proc. Lond. Math. Soc.* **13** (3), 71–89 (1963) [MR0143169 \(26 #729\)](#)
12. Künzi, H.-P.: Nonsymmetric distances and their associated topologies: about the origins of basic ideas in the area of asymmetric topology. In: Aull, C.E., Löwen, R. (eds.) *Handbook of the History of General Topology*, vol. 3, pp. 853–968. Kluwer (2001) [MR1900267 \(2003d:54001\)](#)
13. Naimpally, S.A., Warrack, B.D.: *Proximity Spaces*. Cambridge Tracts in Math, and Math. Phys., vol. 59. Cambridge University Press, Cambridge (1970) [MR0278261 \(43 #3992\)](#)
14. Picado, J.: Weil entourages in pointfree topology. Ph.D. Thesis, University of Coimbra (1995)
15. Picado, J.: Structured frames by Weil entourages. *Appl. Categ. Struct.* **8**, 351–366 (2000) [MR1785854 \(2001e:54051\)](#)
16. Picado, J., Pultr, A.: *Locales Mostly Treated in a Covariant Way*. *Textos de Matemática*, vol. 41, University of Coimbra (2008) [MR2459570 \(2010d:06012\)](#)
17. Picado, J., Pultr, A.: Cover quasi-uniformities in frames. *Topol. Appl.* **158**, 869–881 (2011) [MR2783142 \(2012b:54021\)](#)
18. Picado, J., Pultr, A., Tozzi, A.: Locales. In: Pedicchio, M.C., Tholen, W. (eds.) *Categorical Foundation—Special Topics in Order, Algebra and Sheaf Theory*. *Encyclopedia of Mathematics and its Applications*, vol. 97, pp. 49–101. Cambridge Univ. Press, Cambridge (2004)
19. Pultr, A.: Frames. In: Hazewinkel, M. (ed.) *Handbook of Algebra*, vol. 3, pp. 791–858. Elsevier (2003) [MR2035108 \(2004j:06009\)](#)
20. Schauerte, A.: Biframes. Ph.D. thesis, McMaster University (1992) [MR2689920](#)
21. Schauerte, A.: Biframe compactifications. *Comment. Math. Univ. Carol.* **34** 567–574 (1993) [MR1243088 \(94j:54015\)](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

© Copyright American Mathematical Society 2013