MR3129067 (Review) 06D22 54C30 54D15
Gutiérrez García, Javier (E-EHU-NDM); **Picado, Jorge** (P-CMBR-CM)
Insertion and extension results for pointfree complete regularity. (English summary)
Bull. Belg. Math. Soc. Simon Stevin **20** (2013), *no. 4,* 675–687.

J. Gutiérrez García and the reviewer proved in [Appl. Gen. Topol. **8** (2007), no. 2, 239–242; MR2398514 (2009c:54006)] that complete regularity of a topological space X is equivalent to the following insertion property: If $g, h\colon X \to [0,1]$, h is lower semicontinuous, $g^{-1}[t,1]$ is compact for each $t \in (0,1]$ and $g \le h$, then there exists a continuous $f\colon X \to [0,1]$ such that $g \le f \le h$. The authors of the paper under review analyze to which extent a similar result continues to hold for the case of a completely regular frame L and homomorphisms from the frame of reals into the co-frame of all sublocales of L. They prove that a direct reformulation of the above insertion property is necessary for complete c-regularity of a frame (a new property introduced by the authors which is implied by complete regularity; the authors leave as an open question whether the reverse implication holds) and is sufficient for complete c-regularity of a frame in which every compact sublocale is complemented. The authors also prove that each compact sublocale of a completely regular frame is C-embedded, where the sublocale S is C-embedded if every $f \in C(S)$ has a continuous extension to L. *Tomasz Kubiak*

References

1. R. N. Ball and J. Walters-Wayland, C- and C* -quotients in Pointfree Topol- ogy, Dissertationes Math. (Rozprawy Mat.) **412** (2002) 62pp. MR1952051 (2004a:06007)
2. B. Banaschewski, The Real Numbers in Pointfree Topology, Textos de Matem 'a-tica, Vol. 12, University of Coimbra, 1997. MR1621835 (99c:54017)
3. T. Dube, A little more on Coz-unique frames, Appl. Categ. Structures **17** (2009) 63–73. MR2471248 (2009i:54027)
4. M. J. Ferreira, J. Guti 'errez Garc'ıa and J. Picado, Completely normal frames and real-valued functions, Topology Appl. **156** (2009) 2932–2941. MR2556049 (2011c:54013)
5. J. Guti 'errez Garc'ıa and T. Kubiak, Sandwich-type characterizations of com- pletely regular spaces, Appl. Gen. Topol. **8** (2007) 239–242. MR2398514 (2009c:54006)
6. J. Guti 'errez Garc'ıa and T. Kubiak, General insertion and extension theorems for localic real functions, J. Pure Appl. Algebra **215** (2011) 1198–1204. MR2769226 (2012a:54013)
7. J. Guti 'errez Garc'ıa, T. Kubiak and J. Picado, Localic real-valued functions: a gen- eral setting, J. Pure Appl. Algebra **213** (2009) 1064–1074. MR2498797 (2010a:06020)
8. J. Guti 'errez Garc'ıa and J. Picado, On the algebraic representation of semi- continuity, J. Pure Appl. Algebra **210** (2007) 299–306. MR2319999 (2008b:54025)
9. J. Guti 'errez Garc'ıa and J. Picado, Rings of real functions in pointfree topol- ogy, Topology Appl. **158** (2011) 2264–2278. MR2838376 (2012i:06020)
10. P. T. Johnstone, Stone Spaces, Cambridge Studies in Advanced Mathematics, Vol. 3, Cambridge University Press, Cambridge, 1982. MR0698074 (85f:54002)
11. J. Picado, A new look at localic interpolation theorems, Topology Appl. **153** (2006) 3203–3218. MR2259820 (2007i:06015)
12. J. Picado and A. Pultr, Frames and Locales: Topology without points, Frontiers in

Mathematics, Vol. 28, Springer, Basel, 2012. MR2868166 (2012j:54002)

13. J. J. C. Vermeulen, Some constructive results related to compactness and the (strong) Hausdorff property for locales, in: Category Theory (Proc. Int. Conf. Como, 1990), pp. 401–409, Lecture Notes Math. **1488**, Springer, Berlin, 1991. MR1173026 (93g:54033)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.