MR3149635 (Review) 54G05 06D22 06D30
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On the parallel between normality and extremal disconnectedness. (English summary)
J. Pure Appl. Algebra **218** (2014), *no. 5,* 784–803.

In this article the authors examine the parallel between normality and extremal disconnectedness in the localic/pointfree context. They indicate that the two notions are dual to each other, and determine the source of this duality. In their approach, each pair of results (in this dual layout) is unified under one result with a single proof. Their results are conveniently categorized into four main areas:

(1) Urysohn's separation type lemma,
(2) Tietze's extension type theorem,
(3) Kat ětov-Tong insertion type theorem,
(4) Hausdorff mapping invariance type theorem.

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