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Monotone normality and stratifiability from a pointfree point of view. **(English summary)**
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The somewhat paradoxical sounding title points to the main point of the paper: many classical results in topology may be achieved in a pointfree manner, that is, by a study of the abstract lattices of open (or closed) sets, without involving any points. And moreover, the arguments often become not only more general but also easier than under the constraints of point-set topology. However, unlike other parts of pointfree topology, most of the results in the paper under review heavily lean on choice principles. Specifically, the central concepts under consideration are the monotone or hereditary modifications of (perfectly) normal spaces, extremally disconnected spaces, and stratifiable spaces (in the sense of C. J. R. Borges [Pacific J. Math. **17** (1966), 1–16; MR0188982 (32 #6409)]), an important subclass of monotonically normal spaces. Certain ramifications of basic concepts enter into the picture as soon as the (classically assumed) T_1 axiom is dropped or weakened. In the paper under review, the authors focus on a (pointfreely definable) property, named "subfitness", of J. R. Isbell [Math. Scand. **31** (1972), 5–32; MR0358725 (50 #11184)], and "conjunctivity" of H. Simmons [Proc. Edinburgh Math. Soc. (2) **21** (1978/79), no. 1, 41–48; MR0493959 (58 #12910)] (the dual of "disjunctivity" occurring in Wallman's pioneering work on compactifications). This property is defined by postulating that for $a \not\leq b$ in the given lattice, there exists a c with $a \vee c = 1$ but $b \vee c \neq 1$; applied to topologies, it becomes a weaker axiom than T_1. As usual in order-theoretical considerations, the authors study the relevant properties together with their order-duals; for example, in the pointfree setting, normality is dual to extremal disconnectedness. Several results and their point-set consequences are derived in general for dual frames instead of frames. Furthermore, the authors investigate the stability of some of the properties considered under the formation of open or closed sublocales.

Many of the results in this paper apply to web spaces, defined by the condition that not only the open sets but also the closed sets form a frame [M. Erné, Topology Appl. **156** (2009), no. 12, 2054–2069; MR2532134 (2010g:54017)]. *Marcel Erné*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.