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On permutable pairs of quasi-uniformities. (English summary)

Topology Appl. **196** (2015), *part A,* 260–273.

E. P. de Jager and H.-P. A. Künzi [Topology Appl. **158** (2011), no. 7, 930–938; MR2783147] introduced the notion of permutable quasi-uniformities in the following way: two quasi-uniformities $\mathcal{U}$ and $\mathcal{V}$ on a nonempty set X are permutable if $\mathcal{U}\circ\mathcal{V}=\mathcal{V}\circ\mathcal{U}$. Some results about permutability were proved in that paper, such as the following:

Theorem. Let $\mathcal{P}$ be the Pervin quasi-uniformity of a topological space (X,τ).

(1) $\mathcal{P}\circ\mathcal{P}^{-1}$ is a uniformity if and only if X is normal.

(2) $\mathcal{P}^{-1}\circ\mathcal{P}$ is a uniformity if and only if X is extremally disconnected.

(3) $\mathcal{P}$ and $\mathcal{P}^{-1}$ permute if and only if X is normal and extremally disconnected.

On the other hand, it is known [T. Kubiak, Fasc. Math. No. 19 (1990), 143–145; MR1100180] that normality and extremal disconnectedness are dual notions in lattice-theoretical terms.

In the paper under review, the authors wonder about the possibility of obtaining a framework in which each of the first two statements of the above theorem can be obtained from the other by some kind of dualization process. They show that the necessity of statements (1) and (2) can be proved by dualizing one unique result. Nevertheless, the sufficiency cannot be obtained in this way. Furthermore, they present general notions of normality and disconnectedness which encompass several concepts spread in the literature. This allows them to establish a general context which clarifies the duality between these concepts. *Jesús Rodríguez-López*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.